

## BIR ZARRACHALI SHREDINGER OPERATORI XOS QIYMATI UCHUN ASSIMPTOTIK FORMULALAR

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### ANNOTATSIYA

Ushbu ishda bir o'lchamli panjarada tashqi maydon ta'siri bilan kontakt ta'sirlashuvchi bir kvant zarracha harakatini tavsiflovchi bir zarrachali gamiltonianga mos bir zarrachali diskret Shredinger operatori  $h_\mu$  ning zarrachalar ta'sirlashuv energiyasi  $\mu > 0$  ga bog'liq chapda yoki o'ngda yagona xos qiymatga ega ekanligi isbotlangan.

Ushbu xos qiymat  $z(\mu)$ ,  $\mu > 1$  ning aniq ko'rinishi topilgan. O'zaro ta'sir parametri  $\mu > 1$  ga monoton va uzluksiz bog'liqligi ko'rsatilib, moduli yetarlicha kichik  $\mu > 1$  larda ushbu xos qiymatlar uchun yaqinlashuvchi yoyilmalar olingan.

**Kalit so'zlar:** panjara, operator, xos qiymat, spektr, muhim spektr, yoyilma.

## ASYMPTOTIC FORMULAS FOR EIGENVALUE OF ONE PARTICLE SCHRODINGER OPERATOR

### ABSTRACT

In this study, one particle interaction energy of one particle discrete Schrödinger operator  $h_\mu$  corresponding to a quantum particle gamiltonian describing the motion of a quantum particle in contact with an external field effect in a one-dimensional grid depends on  $\mu > 0$  left or right proved to have a eigenvalue.

This eigenvalue  $z(\mu)$ ,  $\mu > 1$  a clear view of was found. The monotonic and continuous dependence of the interaction parameter on  $\mu > 1$  is shown, and approximate distributions are obtained for these eigenvalues at sufficiently small modulus  $\mu > 1$ .

**Keywords:** lattince, operator, eigenvalue, spectrum, essential spectrum

### KIRISH

Tashqi maydon ta'siridagi bir kvant zarracha harakatini tavsiflovchi bir zarrachali gamiltonianlar hamda qisqa masofada ta'sirlashuvchi ikkita bir xil va har

xil zarrachalar sistemasi gamiltonianlariga mos ikki zarrachali diskret Shredinger operatorlarining spektral xossalari Faria, Corolli, S.Albeverio, S.Laqayevlarning ishlarda o'rganilgan.

Ikki zarrachali diskret Shredinger operatori  $h_\mu(0)$  bir zarrachali gamiltoniani  $h_\mu$  ga unitar ekvivalent va shuning uchun bir zarrachali Shredinger operatorlarining spektral xossalari o'rganish panjaradagi ko'p zarrachali Shredinger operatorlar spektral nazariyasida muhim o'rin tutadi.

## ADABIYOTLAR TAHLILI VA METODOLOGIYA

Ushbu [S.Lakaev, I.Bozorov] ishda uch o'lchamli panjarada potentsialli maydonda tashqi maydon ta'sirida kontakt va bir qadamda ta'sirlashuvga mos bir zarrachali gamiltonian qaralgan. Unga mos diskret Shredinger operatori xos qiymatlar sonining zarrachaning ta'sir energiyalari  $\mu > 0$  va  $\lambda > 0$  larga bog'liqligi to'la o'rganilgan.

B.Saymon va M.Klausning ishida uzluksiz Shredinger operatori xos qiymatlari uchun kichik  $\lambda > 0$  larda  $d = 1$  da yaqinlashuvchi qator,  $d = 2$  da esa kichik  $\lambda > 0$  larda asimptotik yoyilma olingan.

### 2. Bir zarrachali Shredinger operatorinig spektri

$\mathbb{T} = (-\pi, \pi]$  bir o'lchamli tor.  $L_2(\mathbb{T})$  kvadrati bilan integrallanuvchi funksiyalar Gilbert fazosi.  $L^t_2(\mathbb{T}) \subset L_2(\mathbb{T})$  toq funksiyalar qism fazosi bo'lsin.

$L^t_2(\mathbb{T})$  da aniqlangan quyidagi operatorni qaraymiz:

$$h_\mu = h_0 + \mu v,$$

Bu yerda  $h_0$  operator  $\varepsilon(t)$  funksiyaga ko'paytirish operatori, ya'ni

$$(h_0 f)(t) = \varepsilon(t) f(t);$$

$$\varepsilon(t) = 1 - \cos 2t;$$

ta'sir operatori (qo'zg'atish operatori)  $v$  operator esa integral operator bo'lib,  $L^t_2(\mathbb{T})$  da quyidagicha aniqlangan:

$$(vf)(t) = \frac{1}{2\pi} \sin 2t \int_{\mathbb{T}} \sin 2q \cdot f(q) dq;$$

$\mu > 0$  –zarrachaning tashqi muhit bilan ta'sir energiyasi.

**Lemma 1.** Yuqorida aniqlangan  $h_\mu$  operator  $L^t_2(\mathbb{T})$  da o'z-o'ziga qo'shma, chegaralangan operator bo'ladi.

*Isbot.* Hilbert fazosida oʻz-oʻziga qoʻshma operator taʼrifiga koʻra  $A: H \rightarrow H$  operator  $\forall f, g \in H$  uchun  $(Af, g) = (f, Ag)$  shartni qanoatlantirishi kerak.  $(\cdot, \cdot)$  — skalyar koʻpaytma.

Oʻz-oʻziga qoʻshma operator xossasiga koʻra  $h_0$  va  $v$  operatorlarning har birining oʻz-oʻziga qoʻshma operator ekanligini koʻrsatish yetarli.

$$\begin{aligned}(h_0 f, g) &= \int_{\mathbb{T}} (1 - \cos 2q) f(q) \overline{g(q)} dq = \int_{\mathbb{T}} f(q) (1 - \cos 2q) \overline{g(q)} dq = \\ &= \int_{\mathbb{T}} f(q) \overline{(1 - \cos 2q) g(q)} dq = (f, h_0 g); \\ (v f, g) &= \int_{\mathbb{T}} \left( \frac{1}{2\pi} \sin 2q \int_{\mathbb{T}} \sin 2t f(t) dt \right) \cdot \overline{g(q)} dq = \\ &= \int_{\mathbb{T}} f(t) \left( \frac{1}{2\pi} \sin 2t \int_{\mathbb{T}} \sin 2q \cdot g(q) dq \right) dt = (f, v g).\end{aligned}$$

Chegaralanganligi. Operator chegaralanganlik taʼrifidan  $A$  operator uchun  $\exists M > 0$  soni mavjud boʻlib  $\forall f \in D(A) \subset H$  element uchun quyidagi

$$\|(Af)\| \leq M \|f\|;$$

tengsizlik oʻrinli boʻlsa,  $A$  operator chegaralangan deyiladi.

$$\begin{aligned}\|(h_\mu f)(t)\|^2 &= \int_{\mathbb{T}} \left| (1 - \cos 2t) f(t) + \frac{\mu}{2\pi} \sin 2t \int_{\mathbb{T}} \sin 2q f(q) dq \right|^2 dt \leq \\ &\leq 2 \int_{\mathbb{T}} |(1 - \cos 2t) f(t)|^2 dt + 2 \int_{\mathbb{T}} \left| \frac{\mu}{2\pi} \sin 2t \int_{\mathbb{T}} \sin 2q f(q) dq \right|^2 dt \\ &\leq 8 \int_{\mathbb{T}} |f(t)|^2 dt + 2 \int_{\mathbb{T}} \left| \frac{\mu}{2\pi} \int_{\mathbb{T}} \sin 2q f(q) dq \right|^2 dt = \\ &= 8 \int_{\mathbb{T}} |f(t)|^2 dt + \frac{\mu^2}{\pi} \left| \int_{\mathbb{T}} \sin 2q f(q) dq \right|^2 ;\end{aligned}$$

Koshi – Bunyakovski tengsizligiga koʻra:

$$\begin{aligned} \left| \int_{\mathbb{T}} \sin 2q f(q) dq \right|^2 &\leq \int_{\mathbb{T}} \sin^2 2q dq \cdot \int_{\mathbb{T}} |f(q)|^2 dq \\ &= \frac{1}{2} \int_{\mathbb{T}} (1 - \cos 2q) dq \cdot \int_{\mathbb{T}} |f(q)|^2 dq = \pi \|f\|^2; \end{aligned}$$

Demak,  $\|(h_\mu f)(t)\| \leq \sqrt{8 + \pi} \|f\|$ . Lemma isbotlandi. ▲

$v$  – chekli o`lchamli (bir o`lchamli) operator bo`lganidan uning kompakt ekanligiga kelamiz.

$h_\mu$  operator o`z-o`ziga qo`shma bo`lganligi uchun uning qoldiq spektri bo`sh to`plam bo`lib, spektri haqiqiy sonlar to`plamining qismidan iborat, ya`ni

$$\sigma(h_\mu) \subset \mathbb{R}.$$

Quyidagi belgilashlarni kiritamiz:

$$\varepsilon_{\min} = \min_{q \in \mathbb{T}} \varepsilon(q) = 0; \quad \varepsilon_{\max} = \max_{q \in \mathbb{T}} \varepsilon(q) = 2;$$

Muhim spektr turg'unligi haqidagi Veyl teoremasiga ko'ra.  $h_\mu$  operatorning muhim spektri  $\sigma_{\text{ess}}(h_\mu)$  berilgan  $\mu \geq 0$  parametrlardan bog'liq emas va  $h_0$  operatorning spektri bilan ustma-ust tushadi. Shunday qilib

$$\sigma_{\text{ess}}(h_\mu) = \sigma_{\text{ess}}(h_0) = \sigma(h_0) = [\varepsilon_{\min}, \varepsilon_{\max}] = [0, 2]$$

tenglik o`rinli.

## NATIJALAR VA MUHOKAMA

**Teorema 1;**  $\mu > 1$  bo`lsin  $h_\mu$  muhim spektrdan o`ngda yagona xos qiymatga ega va u quyidagi ko`rinishga ega

$$z(\mu) = 2 + \frac{(\mu - 1)^2}{2\mu};$$

Bu xos qiymatga mos xos funksiyaning ko`rinishi.

$$f(q) = -\frac{2\mu^2 C \sin 2q}{\mu^2 + 2\mu \cos 2q + 1}$$

dan iborat.

**Natija.** Quyidagi asimptotik formulalar o`rinli

$$z(\mu) = 2 + \frac{1}{6}(\mu - 1)^2 + o(\mu - 1)^2$$

$$z(\mu) = \frac{\mu}{2} + o\left(\frac{1}{\mu}\right).$$

### 3. $\Delta_\mu(z)$ funksiyani xossalari.

$C$  orqali kompleks sonlar tekisligini belgilaymiz, har bir  $\mu > 0$  va  $z \in C \setminus [0, 2]$  da aniqlangan

$$\Delta_\mu(z) = 1 + \frac{\mu}{2\pi} \int_{\mathbb{T}} \frac{\sin^2 2q}{\varepsilon(q) - z} dq; \quad (1)$$

funksiyani aniqlaymiz.

Bitiruv malakaviy ishini natijasini isbotlashda muhim o'rin tutadigan quyidagi lemmalarni isbotlaymiz.

**Lemma 2.**  $z \in C \setminus [0, 2]$  soni  $h_\mu$  operatorning xos qiymati bo'lishi uchun

$$\Delta_\mu(z) = 0.$$

tenglikning bajarilishi zarur va yetarli.

Isbot; Zarurligi.  $z \in C \setminus [0, 2]$  soni  $h_\mu$  operatorning xos qiymati bo'lsin. U holda

$$(\varepsilon(q) - z)f(q) = -\frac{\mu}{2\pi} \sin 2q \int_{\mathbb{T}} \sin 2tf(t) dt \quad (2)$$

tenglama nolmas  $0 \neq f(q) \in L^2_2(\mathbb{T})$  yechimga ega.

Agar

$$\int_{\mathbb{T}} \sin 2tf(t) dt = C \quad (3)$$

deb belgilash kiritsak. ( $C \neq 0$ )

$$(\varepsilon(q) - z)f(q) = -\frac{\mu}{2\pi} \sin 2q C;$$

tenglikka kelamiz. Bundan  $f(q)$  ni topamiz

$$f(q) = -\frac{\mu C}{2\pi} \frac{\sin 2q}{(\varepsilon(q) - z)}; \quad (4)$$

Ushbu (4) tenglik bilan aniqlangan  $f(q)$  funksiyani (3) belgilashga keltirib qo'yamiz va quyidagi

$$C = -\frac{\mu C}{2\pi} \int_{\mathbb{T}} \frac{\sin^2 2q}{(\varepsilon(q) - z)} dq$$

tenglikka ega bo'lamiz, ya'ni:

$$C + \frac{\mu C}{2\pi} \int_{\mathbb{T}} \frac{\sin^2 2q}{(\varepsilon(q) - z)} dq = 0.$$

(1) belgilashga ko'ra

$$C \Delta_{\mu}(z) = 0.$$

$C \neq 0$  ekanligidan

$$\Delta_{\mu}(z) = 0$$

tenglikka kelamiz.

Yetarliligi.  $\Delta_{\mu}(z) = 0$  bo'lsin. U holda  $z \in C \setminus [0,2]$  soni  $h_{\mu}$  operatorning xos qiymati bo'lishini ko'rsatamiz.

$$f(q) = -\frac{\mu C}{2\pi} \frac{\sin 2q}{(\varepsilon(q) - z)} = -\frac{\mu C}{2\pi} \frac{\sin 2q}{(1 - \cos 2q - z)};$$

funksiyani qurib olamiz. Dastlab  $f(q) \in L_2^t(\mathbb{T})$  bo'lishini ko'rsatamiz.

$$\int_{\mathbb{T}} \left| -\frac{\mu C}{2\pi} \frac{\sin 2q}{(1 - \cos 2q - z)} \right|^2 dq = \left( \frac{\mu C}{2\pi} \right)^2 \int_{\mathbb{T}} \left( \frac{\sin 2q}{(1 - \cos 2q - z)} \right)^2 dq;$$

$z \in C \setminus [0,2]$  ekanligidan  $1 - \cos 2q - z \neq 0$  va  $\frac{\sin 2q}{(1 - \cos 2q - z)}$  — uzluksiz funksiya. Bundan uning chegaralangan ekanligiga kelamiz. Demak,  $f(q) \in L_2(\mathbb{T})$ .

$\sin 2q$  — toq funksiya,  $(1 - \cos 2q - z)$  — juft funksiya. Bundan  $f(q)$  — toq funksiya. Demak,  $f(q) \in L_2^t(\mathbb{T})$ .

Endi bu funksiya  $z$  soni uchun (1) tenglamaning yechimi bo'lishini ko'rsatamiz.

$$\begin{aligned} & (\varepsilon(q) - z) \left( -\frac{\mu C}{2\pi} \frac{\sin 2q}{(1 - \cos 2q - z)} \right) \\ &= -\frac{\mu}{2\pi} \sin 2q \int_{\mathbb{T}} \sin 2t \left( -\frac{\mu C}{2\pi} \frac{\sin 2t}{(1 - \cos 2t - z)} \right) dt. \end{aligned}$$

Ushbu tenglikdan

$$\sin 2q + \frac{\mu}{2\pi} \sin 2q \int_{\mathbb{T}} \frac{\sin^2 2t}{(1 - \cos 2t - z)} dt = 0$$

ga kelamiz, ya'ni:

$$\sin 2q \left( 1 + \frac{\mu}{2\pi} \int_{\mathbb{T}} \frac{\sin^2 2t}{(1 - \cos 2t - z)} dt \right) = 0;$$

(1) belgilashga ko'ra

$$\Delta_{\mu}(z) \sin 2q = 0;$$

$$\Delta_{\mu}(z) = 0.$$

bundan (2) tenglikning o'rinli ekanligiga kelamiz.

Lemma isbot bo'ldi.

**Lemma 2.3.**  $\Delta_\mu(z)$  funksiya  $z \in (2; \infty)$  da monoton o'suvchi bo'lib

$$\lim_{z \rightarrow \infty} \Delta_\mu(z) = 1;$$

$$\lim_{z \rightarrow 2} \Delta_\mu(z) = 1 - \mu.$$

Isbot; (1) formula bilan aniqlangan  $\Delta_\mu(z)$  funksiyadan  $z$  bo'yicha hosila olamiz.

$$\frac{\partial \Delta_\mu(z)}{\partial z} = \mu \int_{\mathbb{T}} \frac{\sin^2 2q}{(\varepsilon(q) - z)^2} dq > 0.$$

Demak,  $\Delta_\mu(z)$  funksiya monoton o'suvchi.

$$\frac{\sin^2 2q}{\varepsilon(q) - z}$$

funksiya  $z > 2$  da uzluksiz va

$$\lim_{z \rightarrow \infty} \frac{\sin^2 2q}{\varepsilon(q) - z} = 0;$$

ekanligidan, integral belgisi ostida limitga o'tish haqidagi teoremaga ko'ra:

$$\lim_{z \rightarrow \infty} \Delta_\mu(z) = 1;$$

ga kelamiz.

$\sin^2 x + \cos^2 x = 1$  ayniyatga ko'ra:

$$\begin{aligned} \lim_{z \rightarrow 2} \Delta_\mu(z) &= \lim_{z \rightarrow 2} \left( 1 + \frac{\mu}{2\pi} \int_{\mathbb{T}} \frac{\sin^2 2q}{\varepsilon(q) - z} dq \right) = 1 + \frac{\mu}{2\pi} \lim_{z \rightarrow 2} \int_{\mathbb{T}} \frac{\sin^2 2q}{\varepsilon(q) - z} dq = \\ &= 1 + \frac{\mu}{2\pi} \int_{\mathbb{T}} \lim_{z \rightarrow 2} \frac{\sin^2 2q}{\varepsilon(q) - z} dq = 1 - \frac{\mu}{2\pi} \int_{\mathbb{T}} \frac{\sin^2 2q}{1 + \cos 2q} dq = \\ &= 1 - \frac{\mu}{2\pi} \int_{\mathbb{T}} \frac{1 - \cos^2 2q}{1 + \cos 2q} dq = 1 - \frac{\mu}{2\pi} \int_{\mathbb{T}} (1 - \cos 2q) dq = 1 - \mu. \end{aligned}$$

Lemma isbotlandi. ▲

Yuqoridagi lemmalardan kelib chiqadiki.  $0 < \mu < 1$  bo'lsa

$$\Delta_\mu(z) = 0$$

tenglama yechimga ega emas.  $\mu > 1$  da faqat bitta yechimga ega bo'ladi.

$\mu = 1$  bo'lsin.

$$\Delta_\mu(z) = 0;$$

tenglama  $z = 2$  yechimga ega.  $z = 2 \in \sigma_{ess}(h_\mu)$ . Bu son  $h_\mu$  operatorning xos qiymat bo'lishini tekshiramiz.

$$(h_{\mu}f)(t) = 2f(t);$$

tenglamani qaraymiz. Bu tenglamadan

$$(1 + \cos 2t)f(t) = \frac{1}{2\pi} \sin 2t \int_{\mathbb{T}} \sin 2qf(q) dq;$$

ga kelamiz.

$$\int_{\mathbb{T}} \sin 2qf(q) dq = C;$$

belgilash olamiz ( $C \neq 0$ ). Bundan

$$f(t) = \frac{C}{2\pi} \cdot \frac{\sin 2t}{1 + \cos 2t};$$

$f(t) \neq 0$ , endi  $f(t) \in L_2^t(\mathbb{T})$  ni tekshiramiz.

$$\begin{aligned} \frac{4\pi^2}{C} \int_{\mathbb{T}} f^2(t) dt &= \int_{\mathbb{T}} \frac{\sin^2 2t}{(1 + \cos 2t)^2} dt = \int_{\mathbb{T}} \frac{1 - \cos 2t}{1 + \cos 2t} dt \\ &= \int_{-\pi}^0 \frac{1 - \cos 2t}{1 + \cos 2t} dt + \int_0^{\pi} \frac{1 - \cos 2t}{1 + \cos 2t} dt = 2 \int_0^{\pi} \frac{1 - \cos 2t}{1 + \cos 2t} dt \geq \end{aligned}$$

(bu xosmas integral bo`lib  $t = \frac{\pi}{2}$  da maxsuslikka ega.)

$$\geq 2 \int_{\frac{\pi}{2}-\delta}^{\frac{\pi}{2}+\delta} \frac{1 - \cos 2t}{1 + \cos 2t} dt \geq C \int_{\frac{\pi}{2}-\delta}^{\frac{\pi}{2}+\delta} \frac{dt}{\left(t - \frac{\pi}{2}\right)^2} = C \int_{-\delta}^{\delta} \frac{dy}{y^2} = 2C_1 \int_0^{\delta} \frac{dy}{y^2}.$$

Ushbu munosabatning o`ng qismida turgan integral uzoqlashuvchi. Demak  $f(t) \notin L_2^t(\mathbb{T})$ .  $z = 2$  xos qiymat emas.

### Asosiy natijalar isboti

#### 2.1 Teoremaning isboti.

Dastlab,

$$\int_{\mathbb{T}} \frac{\sin^2 2q}{\varepsilon(q) - z} dq$$

integralni hisoblaymiz.

$$\int_{\mathbb{T}} \frac{\sin^2 2q}{\varepsilon(q) - z} dq = \int_{\mathbb{T}} \frac{\sin^2 2q}{1 - \cos 2q - z} dq = \int_{\mathbb{T}} \frac{\sin^2 2q}{A - \cos 2q} dq;$$

bu yerda



$$A = 1 - z < -1;$$

Trigonometriyaning asosiy ayniyatidan foydalanib integralni ikki qismga ajratamiz.

$$\begin{aligned} \int_{\mathbb{T}} \frac{\sin^2 2q}{A - \cos 2q} dq &= \int_{\mathbb{T}} \frac{1 - \cos^2 2q}{A - \cos 2q} dq = \int_{\mathbb{T}} \frac{dq}{A - \cos 2q} - \int_{\mathbb{T}} \frac{\cos^2 2q}{A - \cos 2q} dq \\ &= I_1 + I_2; \quad (5) \end{aligned}$$

Birinchi  $I_1$  integralni hisoblaymiz:

$$I_1 = \int_{\mathbb{T}} \frac{dq}{A - \cos 2q};$$

Juft funksiyadan simmetrik oraliq bo'yicha olingan integral xossasidan foydalanib quyidagi tenglikka kelamiz:

$$I_1 = \int_{\mathbb{T}} \frac{dq}{A - \cos 2q} = \int_{-\pi}^0 \frac{dq}{A - \cos 2q} + \int_0^{\pi} \frac{dq}{A - \cos 2q} = 2 \int_0^{\pi} \frac{dq}{A - \cos 2q}.$$

Quyidagicha almashtirish olamiz

$$e^{2iq} = \xi;$$

bundan quyidagilar hosil bo'ladi:

$$\cos 2q = \frac{1}{2} \left( \xi + \frac{1}{\xi} \right),$$

$$d\xi = 2ie^{2iq} dq;$$

$$dq = \frac{d\xi}{2i\xi};$$

Topilganlarni integralga qo'yamiz.

$$\begin{aligned} I_1 &= 2 \int_0^{\pi} \frac{dq}{A - \cos 2q} = 2 \int_{|\xi|=1} \frac{d\xi}{2i\xi \left( A - \frac{1}{2} \left( \xi + \frac{1}{\xi} \right) \right)} = \frac{2}{i} \int_{|\xi|=1} \frac{d\xi}{2A\xi - \xi^2 - 1} \\ &= -\frac{2}{i} \int_{|\xi|=1} \frac{d\xi}{\xi^2 - 2A\xi + 1} = -\frac{2}{i} \int_{|\xi|=1} \frac{d\xi}{(\xi - \xi_1)(\xi - \xi_2)}; \end{aligned}$$

bu yerda

$$\xi_1 = A - \sqrt{A^2 - 1};$$

$$\xi_2 = A + \sqrt{A^2 - 1};$$

Ko'rish mumkinki

$$|\xi_1| > 1, |\xi_2| < 1.$$

Koshining integral formulasi yordamida integralni hisoblaymiz.

$$I_1 = \int_{\mathbb{T}} \frac{dq}{A - \cos 2q} = -\frac{2}{i} \int_{|\xi|=1} \frac{d\xi}{(\xi - \xi_1)(\xi - \xi_2)} = -\frac{2}{i} \cdot 2\pi i \cdot \frac{1}{\xi_2 - \xi_1}$$

$$= -\frac{2\pi}{\sqrt{A^2 - 1}};$$

Demak,

$$I_1 = -\frac{2\pi}{\sqrt{A^2 - 1}}. \quad (6)$$

Endi  $I_2$  integralni hisoblaymiz.

$$I_2 = -\int_{\mathbb{T}} \frac{\cos^2 2q}{A - \cos 2q} dq; \quad (7)$$

Integral ostidagi kasr ifoda suratiga  $A^2$  ni qo`shib ayiramiz va integralni quyidagi ikkita qismga ajratamiz.

$$I_2 = -\int_{\mathbb{T}} \frac{\cos^2 2q}{A - \cos 2q} dq = \int_{\mathbb{T}} \frac{A^2 - \cos^2 2q - A^2}{A - \cos 2q} dq$$

$$= \int_{\mathbb{T}} (A + \cos 2q) dq - A^2 \int_{\mathbb{T}} \frac{dq}{A - \cos 2q} = 2\pi A - A^2 I_1;$$

Demak,

$$I_2 = -\int_{\mathbb{T}} \frac{\cos^2 2q}{A - \cos 2q} dq = 2\pi A + \frac{2\pi A^2}{\sqrt{A^2 - 1}};$$

Lemma 2.4. ga ko`ra  $z \in C \setminus [0,2]$  soni  $h_\mu$  operatorning xos qiymati bo`lishi uchun  $\Delta_\mu(z) = 0$  tenglamani qanoatlantirishi zarur va yetarli. Faraz qilamiz  $z \in C \setminus [0,2]$  xos qiymat bo`lsin. U holda

$$\Delta_\mu(z) = 1 + \frac{\mu}{2\pi} \int_{\mathbb{T}} \frac{\sin^2 2q}{\varepsilon(q) - z} dq = 0;$$

ga ega bo`lamiz. (5), (6) va (7) tengliklarga ko`ra.

$$1 + \frac{\mu}{2\pi} \left( -\frac{2\pi}{\sqrt{A^2 - 1}} + 2\pi A + \frac{2\pi A^2}{\sqrt{A^2 - 1}} \right) = 0;$$

Ushbu tenglik

$$1 + \mu \left( A + \frac{A^2 - 1}{\sqrt{A^2 - 1}} \right) = 0$$

ga ekvivalent. Bundan

$$1 + \mu(A + \sqrt{A^2 - 1}) = 0;$$

$$\mu\sqrt{A^2 - 1} = -1 - \mu A;$$

$$(A < -1, \mu > 1 \text{ bo'lganidan } -1 - \mu A > 0)$$

Tenglamani ikkala tomonini kvadratga ko'taramiz.

$$\mu^2 A^2 - \mu^2 = 1 + 2\mu A + \mu^2 A^2.$$

O'xshash hadlarni qisqartirib soddalashtiramiz, natijada quyidagi A noma'lumga nisbatan chiziqli tenglamani hosil qilamiz:

$$1 + 2\mu A + \mu^2 = 0;$$

$A = 1 - z$  ekanligidan, quyidagi tenglikka kelamiz:

$$2\mu z = 1 + 2\mu + \mu^2;$$

$$z = 2 + \frac{1 - 2\mu + \mu^2}{2\mu};$$

yoki

$$z = 2 + \frac{(1 - \mu)^2}{2\mu}. \quad (8)$$

Bu xos qiymatga mos xos funksiyani aniqlaymiz. (3)ga ko'ra

$$f(q) = \frac{\mu C \sin 2t}{(1 - \cos 2q - z)}$$

Demak, (8) ni hisobga olsak bu xos qiymatga mos xos funksiya:

$$f(q) = -\frac{2\mu^2 C \sin 2q}{\mu^2 + 2\mu \cos 2q + 1}.$$

Ko'rinishda bo'lib bunda  $C = \text{const} \neq 0$ .

Teorema isbotlandi. ▲

## REFERENCES

1. M. Reed and B. Simon: Methods of modern mathematical physics. I: Functional analysis. Academic Press, N.Y., London 1972.
2. V.A.Trenogin: Functional analysis. "Nauka", Moscow 1980.
3. M. Reed and B. Simon: Methods of modern mathematical physics. IV: Analysis of operators. Academic Press, N.Y., 1978.
4. A.N.Kolmogorov, S.V.Fomin: Elementi teorii funktsii i funktsionalno-go analiza. "Nauka", Moscow 1976
5. L.A.Lyusternik, V.I.Sobolev: Elements of functional analysis. "Nau-ka". Moscow 1965

6. N.I.Akhiyezer, I.M.Glazman: Operator theory in Hilbert spaces. "Nau-ka". Moscow 1972
7. P.A.Farva da Veiga, et al. Phys.Rev. E(3) 66, 016130, 9 pp.
8. S.Albeverio, S.N.Lakaev, K.A.Makarov and Z.I.Muminov. The Threshold effects for the two-particle Hamiltonians. Communications in Mathematical Physics, 262, 91-115. (2006).
9. С.Н.Лакаев., И.Бозоров. О спектре одночастичного гамильтониана на трехмерной решетке. ТМФ, 2011г.
10. С.Н.Лакаев., А.М.Халхужаев. Асимптотики собственного значения двух фермионов на одномерной решетке. УзМЖ. 2013г №3.