

MATEMATIKA FANINI O'QITISHDA INNOVATSION TEXNOLOGIYALARDAN VA NOAN'ANAVIY USULLARDAN FOYDALANISH

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ANNOTATSIYA

Ushbu maqolada talabalarni o'qitishning dolzarb masalalariga bag'ishlangan, o'quv jarayoniga innovatsiya tamoyilini tatbiq etishda ijodiy tasavvurni rivojlantirish usullari ochib berilgan. Maqolada bir nechta trigonometrik funktsiyalarni oddiy trigonometrik funktsiyalarga aylantirish uchun noan'anaviy usullardan foydalanish ko'rib chiqiladi.

Kalit so'zlar: Ko'p argumentli trigonometrik funktsiyalar, oddiy trigonometrik funktsiyalar, ikki burchak yig'indisining sinusi, ikki burchak yig'indisining kosinusu, algebraik tenglama, zamonaviy fan va texnika, noan'anaviy usullar, ko'phadli koeffitsientlar, Paskal uchburchagi, sxemalar bilan ishlash.

Kelajak uchun har tomonlama etuk mutaxassis kadrlar tayyorlashning mohiyati, zamonaviy fan va texnikaning rivojlanish talablariga mos barkamol avlodni tarbiyalash masalalari izchillik bilan tashkil etilib, bu boradagi dolzarb masalalar va ularni amalga oshirish chora tadbiri milliy dasturda belgilab berilgan.

Shu ma'noda "trigonometrik funktsiya" larni o'rganishni klassik bo'lmagan (noan'anaviy) usulini misol tariqasida keltirishni lozim topdik.

Algebraik tenglama va funktsiyalarni yaxshi o'rgangan o'quvchilar ham trigonometrik funktsiyalarni o'rganishda ba'zi bir qiyinchiliklarga duch kelishi mumkin. Muammoning asosiy sabablaridan biri, mualliflarning fikricha adabiyotlarning yo'qligida yoki bo'lsa ham juda kamligidadir.

Karrali trigonometrik funktsiyalarni oddiy trigonometrik funktsiyalarga

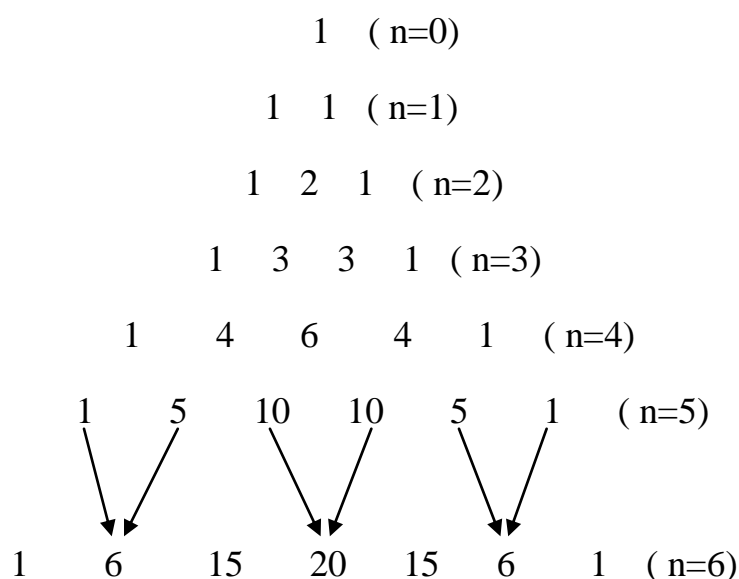
aylantirishda ikki burchak yig'indisining sinusi va kosinuslaridan foydalanadi. Bu an'anaviy usul o'quvchilardan ko'proq vaqt talab qiladi.

Hozirgi fan va texnika taraqqiyoti o'quvchilardan muammolarni tez va o'ta to'g'ri bajarishni talab qiladi. Masalan, $\cos 3x$ karrali funktsiyani oddiy trigonometrik funktsiya ko'rinishida keltiraylik.

$\cos 3x = \cos(x+2x) = \cos x \cos 2x - \sin x \sin 2x$ va ba'zi almashtirishlardan so'ng $4\cos^3 x - 3\cos x$ ifodaga tengligi kelib chiqadi. Bu klassik usul bilan ayniyatni isbotlashda o'quvchilar vaqtni yutqazish bilan birga soddalashtirish jarayonida xatolikka yo'l qo'yishi ham mumkin.

Biz taklif qilgan noan'anaviy usul vaqtni tejash bilan birgalikda algebraik ayniyat bilan trigonometrik funktsiyaning uzviy bog'liqligini asoslaydi.

Ko'phadlarni koeffitsientlarini aniqlashda Paskal uchburchagidan foydalanamiz.



Misol tariqasida $(a + b)^n$ darajasini ko'phad ko'rinishga keltiraylik. Bu hadlarni $a = \cos x$, $b = \sin x$ bilan almashtirsak va $n = 2$ bo'lganda Paskal uchburchagi

$$\begin{aligned}
 (a+b)^2 &= a^2 + 2ab + b^2 \\
 (a+b)^3 &= a^3 + 3a^2b + 3ab^2 + b^3 \\
 (a+b)^4 &= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4
 \end{aligned}$$



$$(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

O'quvchi $\cos nx$ va $\sin nx$ formulalarni oddiy trigonometrik funksiyaga almashtirishning bu usulini quyidagi sxemada ko'rish mumkin (1-sxema):

$$\begin{array}{l} \cos 2x = 1\cos^2 x - 1\sin^2 x \\ + \\ \sin 2x = 2\cos x \sin x \end{array}$$

$$\begin{array}{l} \cos 3x = 1\cos^3 x - 3\cos x \sin^2 x \\ + \\ \sin 3x = 3\cos^2 x \sin x + 1\sin^3 x \end{array}$$

$$\begin{array}{l} \cos 4x = 1\cos^4 x - 6\cos^2 x \sin^2 x - 1\sin^4 x \\ + \\ \sin 4x = 4\cos^3 x \sin x + 4\cos x \sin^3 x \end{array}$$

Bu ma'lumotdan kelib chiqib quyidagi formulalarni taklif qilamiz [1];

$$\cos nx = a_1 \cos^n x - a_3 \cos^3 x - a_5 \sin^2 x + \dots$$

$$\sin nx = a_2 \cos^{n-1} x \sin x - a_4 \cos^{n-3} x \sin^3 x + \dots$$

Bu usul yordamida qiyinroq bo'lgan $tg nx$ funksiyasini ham oddiy funksiya ko'rinishiga keltirishimiz mumkin:

$$\text{Xususiyl holdat } tg 2x = \frac{2tgx}{1-tg^2x}; \quad tg 3x = \frac{3tgx-tg^3x}{1-3tg^2x}; \quad tg 4x = \frac{4tgx-4tg^3x}{1-6tg^2x+tg^4x} \quad \text{va}$$

hakoza.

1- Misol. Ayniyatni isbot qiling.

$$\cos\left(\frac{5}{2}\pi - 6\alpha\right) \sin^3(\pi - 2\alpha) - \cos(6\alpha - \pi) \sin^3\left(\frac{\pi}{2} - 2\alpha\right) = \cos^3 4\alpha$$



Yechish: $\cos\left(\frac{5}{2}\pi - 6\alpha\right) \sin^3(\pi - 2\alpha) - \cos(6\alpha - \pi) \sin^3\left(\frac{\pi}{2} - 2\alpha\right) =$

$$= \cos\left(\frac{5}{2}\pi - 6\alpha\right) (\sin(\pi - 2\alpha))^3 - \cos(\pi - 6\alpha) \left(\sin\left(\frac{\pi}{2} - 2\alpha\right)\right)^3 =$$

$$= [\sin 3x = 3\sin x - 4\sin^3 x, \cos 3x = 4\cos^3 x - 3\cos x] = \sin 6\alpha \sin^3 2\alpha +$$

$$+ \cos 6\alpha \cos^3 2\alpha = \sin 3(2\alpha) \sin^3 2\alpha + \cos 3(2\alpha) \cos^3 2\alpha = (3\sin 2\alpha -$$

$$- 4\sin^3 2\alpha) \sin^3 2\alpha + (4\cos^3 2\alpha - 3\cos 2\alpha) \cos^3 2\alpha = 3\sin^4 2\alpha - 4\sin^6 2\alpha +$$

$$+ 4\cos^6 2\alpha - 3\cos^4 2\alpha = -3(\cos^4 2\alpha - \sin^4 2\alpha) + 4(\cos^6 2\alpha - \sin^6 2\alpha) =$$

$$= -3(\cos^2 2\alpha - \sin^2 2\alpha)(\cos^2 2\alpha + \sin^2 2\alpha) + 4(\cos^2 2\alpha - \sin^2 2\alpha) \times$$

$$\times (\cos^4 2\alpha + \cos^2 2\alpha \sin^2 2\alpha + \sin^4 2\alpha) = -3(\cos^2 2\alpha - \sin^2 2\alpha) +$$

$$4(\cos^2 2\alpha - \sin^2 2\alpha)((\cos^4 2\alpha + \sin^4 2\alpha) + \cos^2 2\alpha \sin^2 2\alpha) = -3\cos 4\alpha +$$

$$4\cos 4\alpha((\cos^2 2\alpha + \sin^2 2\alpha)^2 - 2\cos^2 2\alpha \sin^2 2\alpha + \cos^2 2\alpha \sin^2 2\alpha) =$$

$$-3\cos 4\alpha + 4\cos 4\alpha(1 - \cos^2 2\alpha \sin^2 2\alpha) = -3\cos 4\alpha + 4\cos 4\alpha - \cos 4\alpha \times$$

$$\times (4\sin^2 2\alpha \cos^2 2\alpha) = \cos 4\alpha - \cos 4\alpha(4\sin^2 2\alpha \cos^2 2\alpha) = \cos 4\alpha - \cos 4\alpha \times$$

$$\sin^2 4\alpha = \cos 4\alpha(1 - \sin^2 4\alpha) = \cos 4\alpha \cos^2 4\alpha = \cos^3 4\alpha.$$

2- misol. Tenglamani yeching.

$$2 \cos 13x + 3 \cos 3x + 3 \cos 5x - 8 \cos x \cos^3 4x = 0$$

Yechish. $\cos 3\alpha = 4\cos^3 \alpha - 3\cos \alpha$, $\cos \alpha + \cos \beta = 2\cos \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2}$,

$$\cos \alpha - \cos \beta = 2\sin \frac{\alpha+\beta}{2} \sin \frac{\beta-\alpha}{2}$$

$$2\cos 13x + 3(\cos 3x + \cos 5x) - 8\cos x \cos^3 4x = 0$$

$$\Leftrightarrow 2\cos 13x + 3 \cdot 2\cos 4x \cos x - 8\cos x \cos^3 4x = 0 \Leftrightarrow$$

$$\Leftrightarrow 2\cos 13x - 2\cos x(4\cos^3 4x - 3\cos 4x) = 0 \Leftrightarrow$$

$$\Leftrightarrow 2\cos 13x - 2\cos x \cos 12x = 0 \Leftrightarrow 2\cos 13x - (\cos 13x + \cos 11x) = 0 \Leftrightarrow$$

$$\Leftrightarrow -2\sin 12x \sin x = 0$$

$$1) \sin 12x = 0, 12x = \pi k, x_1 = \frac{\pi k}{12}, k \in \mathbb{Z}$$

$$2) \sin x = 0, x_2 = \pi n, n \in \mathbb{Z} \text{ chet ildiz}$$

$$\text{Javob. } x = \frac{\pi k}{12}, k \in \mathbb{Z}$$

Shunday qilib, karrali trigonometrik funksiyalarni oddiy trigonometrik funksiyaga aylantirishning noan'anaviy va an'anaviy usulga qaraganda bir qancha qulayliklarga ega. Shulardan biri vaqtni tejash bilan birga o'quvchilarda sxemalar bilan ishlash mahoratini shakllantiradi. Shuningdek, trigonometrik funksiyalarni algebraik ayniyatlar bilan uzviy bog'liqligini ta'minlaydi. Bir vaqtda karrali trigonometrik funksiyalarni ayni $\cos nx$, $\sin nx$ larni oddiy trigonometrik funksiyalarga o'tkazish mumkin. (1-sxema)

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