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IKKI O'LCHOVLI YEVKLID FAZOLARIDA KILLING VEKTOR MAYDONLAR GEOMETRIYASI

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ANNOTATSIYA

Mazkur maqolada ikki o'lchovli Yevklid fazosida Killing vektor maydonlar geometriyasi o'rganilgan. Ikki o'lchovli Yevklid fazosida aniqlandan ixtiyoriy Killing vektor maydonning umumiy ko'rinishi topilgan. Bundan tashqari umumiy ko'rinishdagi Killing vektor maydon integral chizig'i va oqimi topilgan.

Kalit so'zlar: Yevklid tekisligi, vektor maydon, Killing vektor maydoni, vektor maydon integral chizig'i, vektor maydon oqimi, differensial operator, infinitesimal yasovchi, differensial tenglama.

ABSTRACT

This article considers the geometry of Killing vektor fields in two-dimensional Euclidean space. The general form of an arbitrary Killing vektor field in two-dimensional Euclidean space is found. In addition, an integral curve and the flow of the Killing vektor field given in the general form are found.

Keywords: Euclidean plane, vector field, Killing vector field, integral line of vector field, flow of vector field, differential operator, infinitesimal constructor, differential equation.

KIRISH

Ta'rif 1. R^2 fazoning har bir $M(x_1, x_2)$ nuqtasiga $\{\xi_1, \xi_2\}$ vektor mos qo'yuvchi $X(x_1, x_2) \to \{\xi_1, \xi_2\}$ akslantirishga vektor maydon deyiladi.

Vektor maydonni $X = \{\xi_1, \xi_2\}$ ko'rinishda yoki differensiallanuvchi operator sifatida,

$$X = \xi_1 \frac{\partial}{\partial x_1} + \xi_2 \frac{\partial}{\partial x_2}$$

ko'rinishida belgilanishi mumkin.

R^2 fazoda $\gamma(t) = \big(x_1(t), x_2(t)\big)$ chiziq berilgan bo'lsin. U holda bu chiziqning har bir nuqtasida ikkita vektor, ya'ni vektor maydonning shu nuqtadagi vektori va egri chiziqning shu nuqtadagi urunma vektori mavjud bo'ladi.

Ta'rif 2. Agar barcha $t = t_0$ lar uchun $X\big(\gamma(t_0)\big) = \gamma'(t_0)$ tenglik o'rinli bo'lsa, $\gamma(t)$ chiziq X vektor maydonning $\gamma(t_0)$ nuqtadan o'tuvchi integral chizig'i deyiladi.

Agar vektor maydon silliq, ya'ni ξ_1, ξ_2 funksiyalar silliq bo'lsa, fazoning har bir nuqtasidan yagona integral chiziq o'tadi. Natijada quyidagi akslantirishlar oilasini hosil qilamiz: $X^t: (x_1, x_2) \to \big(x_1(t), x_2(t)\big)$. $X^t - X$ vektor maydon oqimi deyiladi.

Ta'rif 3. Agar X vektor maydon oqimi harakatlar gruppasini hosil qilsa, bu vektor maydon Killing vektor maydoni deyiladi.

Misol 1. $X = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}$ ushbu vektor maydon oqimi, $X^t(x, y) = (xcost - ysint, xsint - ycost)$ dan iborat. Bu vektor maydon oqimi, koordinatalar boshiga nisbatan burishlardan iborat, harakatlar gruppasini hosil qiladi. Shuning uchun $X = -y\frac{\partial}{\partial x} + x\frac{\partial}{\partial y}$ vektor maydon Killing vektor maydoniga misol bo'la oladi.

Quyidagi ma'lum teoremani keltiramiz [1].

Teorema-1. $X = \xi\frac{\partial}{\partial x} + \eta\frac{\partial}{\partial u}$ vektor maydon Killing vektor maydon bo'lishi uchun

$$\frac{\partial\xi}{\partial x} = \frac{\partial\eta}{\partial y} = 0;$$

va

$$\frac{\partial\xi}{\partial y} + \frac{\partial\eta}{\partial x} = 0$$

tengliklarning bajarilishi zarur va yetarli.

Bu teorema yordamida tekislikda berilgan vektor maydonning Killing vektor maydon bo'lishi yoki bo'lmasligini osongina tekshirib ko'rish mumkin.

Misol 2. Ushbu $X = (2021y + 2022)\frac{\partial}{\partial x} + (2020 - 2021x)\frac{\partial}{\partial y}$ vektor maydonni Killing vektor maydon bo'lish yoki bo'lmasligini tekshirib ko'raylik.

$$\frac{\partial(2021y + 2022)}{\partial x} = \frac{\partial(2020 - 2021x)}{\partial y} = 0,$$

$$\frac{\partial(2021y + 2022)}{\partial y} + \frac{\partial(2020 - 2021x)}{\partial x} = 2021 - 2021 = 0.$$

Demak, yuqoridagi 1-teoremaga asosan $X = (2021y + 2022)\frac{\partial}{\partial x} + (2020 - 2021x)\frac{\partial}{\partial y}$ vektor maydon Killing vektor maydon bo'ladi.

ADABIYOTLAR TAHLILI VA METODOLOGIYA

Killing vektor maydon oqimi hosil qilgan akslantirishlar nuqtalar orasidagi masofani saqlaydi. Shuning uchun harakatlar gruppalarining infenitizimal yasovchilari Killing vektor maydonlar bo'ladi. Vektor maydonlar geometriyasi ko'plab ishlarda o'rganilgan [1-10].

A.Ya.Narmanov, J.Aslonovlar ishlarida Killing vektor maydonlar oilalari orbitalari hosil qilubchi qatlamalarning klassifikatsiyasi o'rganilgan [8]. A.Ya.Narmanov, S. Saitovalar ishlarida Killing vektor maydonlar oilasi to'la integrallanuvchi bo'lishining zaruriy va yetarli shartlari keltirilgan. A.Ya.Narmanov, O.Qosimovlar ishlarida vector maydonlar oilalari hosil qulivchi singulyar qatlamalar geometriyasi o'rganilgan [7].

Kompleks analiz kursidan ma'lumki, $z(x, y) = u + iv$ funksiyaning haqiqiy va mavhum qismlari uchun quyidagi

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}; \qquad (1)$$

va

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 0 \qquad (2)$$

tengliklar o'rinli bo'lsa, $z(x, y) = u + iv$ funksiya golomorf funksiya deyiladi. Golomorf funksiyaning shartlari bilan 1-teorema shartlarida o'xshashliklar bor. Golomorf funksiyalar to'plamidan bir qismini ajratib olsak Kiiling vektor maydonlarini hosil qilishimiz mumkin. Buning uchun qo'shimcha yana bir shart qo'yamiz:

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 0;$$

Bu tengliklardan ko'rinib turibdiki, u funksiya x erkin o'zgaruvchiga bog'liq emas, ya'ni bu funksiya faqatgina y erkin o'zgaruvchining funksiyasi bo'ladi, demak $u = u(y)$. Xuddi shuningdek, v funksiya y erkin o'zgaruvchiga bog'liq emas, ya'ni bu funksiya faqatgina x erkin o'zgaruvchining funksiyasi bo'ladi, demak $v = v(x)$. Yuqoridagi (2) teklikni hisobga olsak, quyidagi

$$\frac{du}{dy} = -\frac{dv}{dx}$$

tenglikni hosil qilamiz. Bu tenglikdan $\frac{du}{dy} = -\frac{dv}{dx} = const$ ekanligi kelib chiqadi. Chunki bu ifoda o'zgarmas funksiyadan boshqa biror funksiyaga teng bo'lsa, $u = u(y)$ yoki $v = v(x)$ ekanligiga zid bo'lib qoladi. Shuning uchun,

$$\frac{du}{dy} = -\frac{dv}{dx} = c$$

deb olib, $u = -cy + a$ va $v = cx + b$ tengliklarni hosil qilamiz. Demak, tekislikda, $z(x,y) = (-cy + a\,) + i(cx + b\,)$ ko'rinishdagi golomorf funksiya Killing vektor maydonni hosil qiladi.

Demak, keltirilganlarga tayangan holda quyidagi teoremani hosil qilamiz.

Teorema-2. Ikki o'lchovli Yevklid fazosida ixtiyoriy Killing vektor maydon ko'rinishi

$$X = (-cy + a)\frac{\partial}{\partial x} + (cx + b)\frac{\partial}{\partial y} \qquad (3)$$

kabi bo'ladi.

Agar $c = 0$ bo'lsa, $X = a\frac{\partial}{\partial x} + b\frac{\partial}{\partial y}$ vektor maydon Killing vektor maydon bo'lib, uning oqimi, $\{a, b\}$ vektor bo'ylab parallel ko'chirishlardan iborat bo'ladi. Agar $a = b = 0$ bo'lsa, $X = -cy\frac{\partial}{\partial x} + cx\frac{\partial}{\partial y}$ vektor maydon Killing vektor maydon bo'lib, uning oqimi, koordinata boshiga nisbatan burishlardan iborat bo'ladi.

Endi (3) vektor maydonning integral chizig'ini $c \neq 0$ holda topamiz. Buning uchun ta'rifga ko'ra ushbu

$$\begin{cases} \dot{x} = -cy + a \\ \dot{y} = cx + b \end{cases} \qquad (4)$$

differensial tenglamalar sistemasini yechamiz:

$$\begin{cases} cy = -\dot{x} + a \\ c\dot{y} = -\ddot{x} \end{cases}$$

$$c\dot{y} = c^2x + bc$$

$$-\ddot{x} = c^2x + bc$$

Buning uchun quyidagi ikkinchi tartibli bir jinsli bo'lmagan, chiziqli differensial tenglamani yechamiz:

$$\ddot{x} + c^2x + bc = 0. \qquad (5)$$

Bu ikkinchi tartibli bir jinsli bo'lmagan, chiziqli differensial tenglamani yechish uchun unga mos bir jinsli ikkinchi tartibli, chiziqli differensial tenglamani yechib olamiz:

$$\ddot{x} + c^2x = 0,$$

$$x = e^{kt},$$
$$\dot{x} = ke^{kt},$$
$$\ddot{x} = k^2e^{kt},$$
$$k^2e^{kt} + c^2e^{kt} = 0,$$
$$k^2 + c^2 = 0,$$
$$k = \pm ci,$$
$$x = C_1e^{cit} + C_2e^{c(-it)} = cosct(C_1 + C_2) + sinct(C_1 - C_2).$$

Endi (5) ikkinchi tartibli bir jinsli bo’lmagan, chiziqli differensial tenglamani yechimini topamiz:

$$x = cosct(C_1 + C_2) + sinct(C_1 - C_2) + A,$$
$$c^2A = -bc,$$
$$A = -\frac{b}{c},$$
$$x = (C_1 + C_2)cosct + (C_1 - C_2)sinct - \frac{b}{c}.$$

Hosil qilingan yechim va (4) sistemaning birinchi tenglamasi yordamida quyidagilarni hosil qilamiz:

$$Cy = -C(C_1 + C_2)sinct + C(C_1 - C_2)cosct + a,$$
$$y = (C_1 + C_2)sinct - (C_1 - C_2)cosct + \frac{a}{c}.$$

Demak, (4) differensial tenglamalar sistemasini umumiy yechimi quyidagi ko’rinishda bo’ladi

$$\begin{cases} x = (C_1 + C_2)cosct + (C_1 - C_2)sinct - \frac{b}{c} \\ y = (C_1 + C_2)sinct - (C_1 - C_2)cosct + \frac{a}{c} \end{cases}.$$

Endi quyidagi boshlang’ich shartlar

$$\begin{cases} x(0) = x_0 \\ y(0) = y_0 \end{cases}$$

ni hisobga olgan holda integral chiziqni hosil qilamiz:

$$\begin{cases} x_0 = C_1 + C_2 - \frac{b}{c} \\ y_0 = -C_1 + C_2 + \frac{a}{c} \end{cases}$$

$$x_0 - y_0 = 2C_1 - \frac{a + b}{c},$$
$$x_0 + y_0 = 2C_2 + \frac{a - b}{c},$$

$$C_1 = \frac{x_0 - y_0}{2} + \frac{a+b}{2c},$$
$$C_2 = \frac{x_0 + y_0}{2} - \frac{a-b}{2c},$$
$$x = \left(\frac{x_0 - y_0}{2} + \frac{a+b}{2c} + \frac{x_0 + y_0}{2} - \frac{a-b}{2c}\right) cosct + \left(\frac{x_0 - y_0}{2} + \frac{a+b}{2c} - \frac{x_0 + y_0}{2} + \frac{a-b}{2c}\right) sinct - \frac{b}{c} = \left(x_0 + \frac{b}{c}\right) cosct - \left(y_0 - \frac{a}{c}\right) sinct - \frac{b}{c},$$
$$y = \left(\frac{x_0 - y_0}{2} + \frac{a+b}{2c} + \frac{x_0 + y_0}{2} - \frac{a-b}{2c}\right) sinct - \left(\frac{x_0 - y_0}{2} + \frac{a+b}{2c} - \frac{x_0 + y_0}{2} + \frac{a-b}{2c}\right) cosct + \frac{a}{c} = \left(x_0 + \frac{b}{c}\right) sinct + \left(y_0 - \frac{a}{c}\right) cosct + \frac{a}{c}.$$

NATIJA

Demak, (3) vektor maydonning (x_0, y_0) nuqtadan o'tuvchi integral chizig'i quyidagi

$$\begin{cases} x(t) = \left(x_0 + \frac{b}{c}\right) cosct - \left(y_0 - \frac{a}{c}\right) sinct - \frac{b}{c} \\ y(t) = \left(x_0 + \frac{b}{c}\right) sinct + \left(y_0 - \frac{a}{c}\right) cosct + \frac{a}{c} \end{cases} \quad (6)$$

parametrik tenglama yordamida aniqlanadi.

(3) vektor maydonning (x_0, y_0) nuqtadan o'tuvchi integral chizig'i qanday chiziq aniqlashini topaylik. Buning uchun (6) sistemada t parametrni yo'qotamiz.

$$\begin{cases} x + \frac{b}{c} = \left(x_0 + \frac{b}{c}\right) cosct - \left(y_0 - \frac{a}{c}\right) sinct \\ y - \frac{a}{c} = \left(x_0 + \frac{b}{c}\right) sinct + \left(y_0 - \frac{a}{c}\right) cosct \end{cases}$$

Bu tengliklarni kvadratga ko'tarib qo'shamiz:

$$\left(x + \frac{b}{c}\right)^2 + \left(y - \frac{a}{c}\right)^2 = \left(\left(x_0 + \frac{b}{c}\right) cosct - \left(y_0 - \frac{a}{c}\right) sinct\right)^2 +$$
$$+ \left(\left(x_0 + \frac{b}{c}\right) sinct + \left(y_0 - \frac{a}{c}\right) cosct\right)^2 = \left(x_0 + \frac{b}{c}\right)^2 cos^2ct -$$

$$-2\left(x_0+\frac{b}{c}\right)\left(y_0-\frac{a}{c}\right)sinctcosct+\left(y_0-\frac{a}{c}\right)^2 sin^2ct+\left(x_0+\frac{b}{c}\right)^2 sin^2ct+$$

$$+2\left(x_0+\frac{b}{c}\right)\left(y_0-\frac{a}{c}\right)sinctcosct+\left(y_0-\frac{a}{c}\right)^2 cos^2ct=$$

$$=\left(x_0+\frac{b}{c}\right)^2(cos^2ct+sin^2ct)+\left(y_0-\frac{a}{c}\right)^2(cos^2ct+sin^2ct)-$$

$$-2\left(x_0+\frac{b}{c}\right)\left(y_0-\frac{a}{c}\right)sinctcosct+2\left(x_0+\frac{b}{c}\right)\left(y_0-\frac{a}{c}\right)sinctcosct=$$

$$=\left(x_0+\frac{b}{c}\right)^2+\left(y_0-\frac{a}{c}\right)^2.$$

Demak, (3) vektor maydonning (x_0, y_0) nuqtadan o'tuvchi integral chizig'i markazi $\left(-\frac{b}{c},\frac{a}{c}\right)$ nuqtada, radiusi $r=\sqrt{\left(x_0+\frac{b}{c}\right)^2+\left(y_0-\frac{a}{c}\right)^2}$ ga teng bo'lgan aylanadan iborat ekan.

Misol 3. $X=(-4y+16)\frac{\partial}{\partial x}+(4x+12)\frac{\partial}{\partial y}$ vektor maydonning (0,0) nuqtadan o'tuvchi integral chizig'ini toping.

Yechish. (6) dan foydalanib berilgan vektor maydonning (x_0, y_0) nuqtadan o'tuvchi integral chizig'ini quyidagicha yoza olamiz:

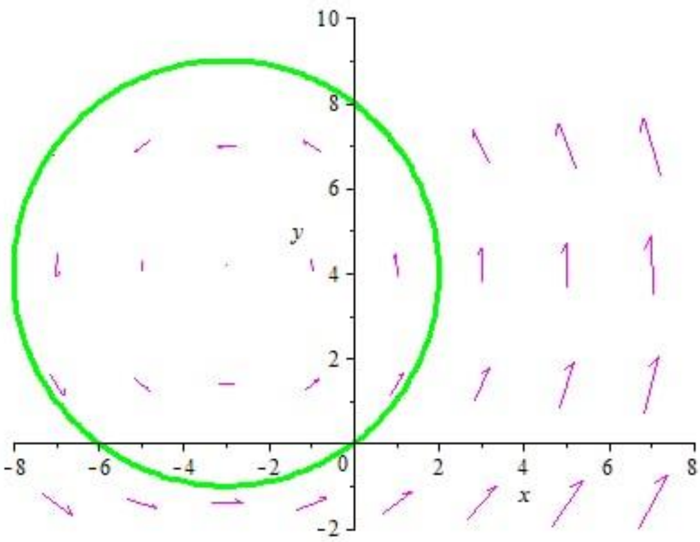
$$\begin{cases} x(t)=(x_0+3)cosct-(y_0-4)sinct-3 \\ y(t)=(x_0+3)sinct+(y_0-4)cosct+4 \end{cases}.$$

Yuqorida keltirilganlarni hisobga olib, berilgan vektor maydonning (x_0, y_0) nuqtadan o'tuvchi integral chizig'i, markazi $(-3,4)$ nuqtada, radiusi $r=\sqrt{(x_0+3)^2+(y_0-4)^2}$ ga teng bo'lgan aylana ekanligini hosil qilamiz.

Shulardan xulosa qiladigan bo'lsak, $X=(-4y+16)\frac{\partial}{\partial x}+(4x+12)\frac{\partial}{\partial y}$ vektor maydonning (0,0) nuqtadan o'tuvchi integral chizig'i markazi $(-3,4)$ nuqtada, radiusi $r=5$ ga teng bo'lgan aylanadan iborat bo'ladi.

XULOSA VA TAKLIFLAR

Ushbu maqolada Yevklid tekisligidagi ixtiyoriy Killing vektor maydonning umumiy ko'rinishi topilgan. Tekislikda umumiy ko'rinishda berilgan Killing vektor maydonlarining integral chiziqlari va oqimlari topilgan. Demak, ikki o'lchamli Yevklid tekisligida Killing vektor maydonlarnining klassifikatsiyasini hosil qildik. Bu natijalardan geometriya ixtisosligi magistr talabalarining maxsus kurslarida va ilmiy ishlarda foydalanish mumkin.



Arrows of the vector field, and the flow line(s) emanating from the given initial point(s)

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