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PREDICTION OF ELECTRIC VEHICLES CHARGING: A (MAX, +) ALGEBRA-BASED APPROACH

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ABSTRACT

Nowadays, one of the most major issues for electric vehicles (EVs) concerns their charging. In our previous research works in this field, we have proposed a predictive charging policy aiming to reduce waiting times of EVs within charging stations. In this paper we continue this effort by developing a formal model to describe and study the charging process behavior. Based on this model, the objective consists of serving a large number of charging demands with a minimum of time tacking into account many parameters of the system such as the location of EVs, their consumption and their real needs in terms of energy. Our developed model is described as states/transitions occurring sequence in a chronological order. It based on Petri net model in which each event execution requires the time amount. Managing charging demands is ensured by a collaborative platform based on communication technologies and geopositioning techniques. The behavior of this platform is represented and access control of registered EVs is also addressed. Some qualitative properties of charging system such as reachability, deadlock-free and synchronization are checked and validated using Petri net theory. Afterwards, we combine Petri nets with (Max, +) equations in order to express the analytic behavior of the process and, to analyze and validate some of its quantitative properties.

Keywords: Electric vehicles, Charging process, Predictive charging, Modeling, Evaluation, Petri nets, (Max, +) algebra.

1. INTRODUCTION

Charging electric vehicles with a minimum time while keeping a maximum of performances they provide is the current challenge of constructors and researches in the field. Among these performances we underline traveling long distances (autonomy) and unconditionally using on board services (air conditioning, radio, lighting, etc.). Moreover, the limited cruising range and the ability to regain energy during deceleration are the main features of EVs. In this case, to suggest the nearest charging station is not enough. However, guiding vehicles into adequate charging stations could be effective and avoid serious problems such as long waiting and breakdown. In our recent research work subject to the EVs charging management [14] which was addressed one of the most major issues related to the drivers uncertainty to get a suitable and vacant place at a charging station. We continue these efforts and try to propose a formal approach in order to anticipate, plan and propose the EVs adequate charging solutions in our current work. The studied system can be seen as a discrete Event System (DES) managed by $State/Transition$ concept. In this system a $State$ represents, for example, charging operation of an EV, waiting for charging, traveling before/after charging, etc.; and a $Transition$ models beginning/ending dates of each process state. It is worth noting that a transition may include the following optional features: event, a guard condition or a list of actions. In our current work a transition is an event signifying the transition form a state of the system to another one. Considering the chronological evolution of charging operations, the problem will be then modeled by a well-known powerful formalism already developed for DES. To do so, we propose a unified methodology based on Petri nets (PN) combined with a mathematical algebra, called dioïd algebra, to model and evaluate the performances of the system using linear equations. Both formalisms have specific advantages they present. Furthermore, using these two complementary formalisms in order to model and analyze synchronized and concurrent systems is effective as well. More precisely, as a design language for the complex processes specification can be used PN and its theory provides powerful analysis techniques with mathematical background which can be used to verify the modelled systems correctness. Many sub-classes of PN are developed in the literature according to the studied system. For DES, Event Graphs (EG) are more suitable to address the phenomena such as synchronization, parallelism and concurrency. When the system behavior evolves over time, Timed Event Graphs (TEG) are a well-founded process modeling technique integrating the time factor for the evaluation of delays and beginning/ending dates of each process state. These extensions of PN facilitate the modeling of complex processes where time and data are important factors. Combined with TEG, dioid algebra provides a complete analysis of studied systems by studying and verifying some quantitative properties of the system. In addition this algebra is will-known as powerful formal fool for describing the behavior of systems characterized by delays and synchronization. In this paper we mainly use the (max, +) algebra for calculating the occurrence date, called also dater, of each system event. Among research works developed in the literature focusing on these tools, there are [2],[5], [10] and [11].

Using these formalism, the proposed models in this paper describe the behavior of a developed collaborative platform for charging process for which the architecture is detailed in [14]. Furthermore, the access control to provided services by the platform (e.g. suggestion of a suitable charging station for an EV, suggestion of a charging station with other interest points such as shopping, restaurant, etc.) is addressed by the proposed models. TEG will be used to prove graphically, and also analytically, some properties (reachability, deadlock, etc.) of the charging system and to evaluate some performance measures (waiting times, response times, occupation times, etc.). However, TEG model cannot offer the possibility to ensure a complete study of the proposed process. To remedy to this lack, we combine the obtained TEG model with a state representation in dioid algebra in order to describe its behavior with mathematical linear equations thereby allowing to reach suitable and convincing results. Through a case study, allying TEG with dioid algebra is not only a powerful methodology for specification and modeling will be shown in this paper and adequate tool for behavior prediction and decision-making as well.

The remainder of this paper is organized as follows. The Section 2 represents a survey of related work. Our research context is given in Section 3. In Section 4, the problem statement and modeling approach are presented. Section 5 presents how the system sizing can be carried out for performance tuning and improvement. The Section 6 concludes the paper and gives some future directions of this work.

2. RELATED WORK

With the promotion of EVs many challenges are to be addressed such as charging and discharging process which can be a serious source of troubles. In fact, to charge their EVs drivers have no solutions except to put their EVs under charging for several hours. To remedy to this problem of long waiting, many researches have been conducted in the literature and try to propose some solutions and alternatives. Otherwise, many researches have been conducted to schedule, control and optimize charging and discharging processes of EVs. In what follows we present some of these research works. Several methods and architectures for charging/discharging of EVs have been proposed. For example, in [4] a distributed architecture is proposed. This architecture achieves energy balancing at three levels: at the charging station level through controlled charging and at two other levels. The aim of this work is to create guidance EV users to a charging station in order to charge optimally by using routing server. This server enables to dispatch the reservations on the basis of the rough availability of resources at certain selected charging stations.

In the same context, the authors of [13] and [8] proposed some approaches for effective planning charging times. In [8] is given a reservation-based scheduling scheme, which is to decide the multiple requests service order for the charging station. The objective of this scheme is to improve the drivers' satisfiability of EVs to reduce the charge costs and waiting times. In [18], an agent-based approach is developed to represent and control the behavior of charging and discharging operations of Plug-in Hybrid Electric Vehicles (PHEV). A comparative study was proposed in this work about reactive scheduling and proactive scheduling for reducing imbalance costs. Some simulations were worked out and obtained results show a reduction of imbalance costs by 14% with reactive scheduling. Contrariwise,using proactive scheduling the imbalance cost reduction is about 44%. The work presented in [16] focused on an optimization of charge pattern problem of a PHEV. The objective is double: minimizing the costs of fuel and electricity and the degradation of the battery health over a 24 hours drive cycle. A stochastic optimization method is used to reach the first objective, whereas an electrochemistry-based model is adopted to reach the second objective. This model is based on an anode-side res istive film formation in lithium-ion batteries.

A genetic algorithm used for optimizing the charging behavior of a PHEV was proposed in [9]. Trough this study, the authors aim to maximize the energy trading profit in a V2G context, and second minimize the battery cost. Furthermore, the charging and discharging processes have been addressed in [7] as a scheduling optimization problem. In this study, charging power is considered to minimize the total cost of EVs. Other charging algorithms are developed respectively in [21], [3] for fast charging and increasing battery life cycles. In [6] authors have addressed the issue of management of parking lots and charging station management systems. They have proposed to manage the charging process within parking lots by using sensors which detect whether a parking space is free or occupied. Furthermore, an existing luminary within charging stations guides customers in search of a parking space.

Regarding the combination of TEG and (max, +) algebra, the proposed modeling approach in this paper is based on what the authors have proposed and developed in [11]. Allying TEG with (max, +)-algebra is proposed to describe and analyze the electronic signature process of contracts between clients and server to answer clients requests. The objective of the study is to serve a maximum of requests using a minimum of servers. We make then an analogy between the problem addressed in this paper in the proposed approach in in [11], and the objective is to serve a maximum charging requests using a minimum charging stations without excessive waiting of drivers.

3. THE CONTEXT OF RESEARCH: MANAGEMENT OF EVS CHARGING

According to the electric power and the extent of energy conservation, three technology types of EVs are developed. Fully electric vehicles (FEV), Hybrid electric vehicles (HEV), and plug-in hybrid electric vehicles (PHEV). The first technology FEV are primarily suited for short journeys with limited ranges. However, the duration of their charging processes may take several hours. In this technology, batteries have to be efficiently used since the propulsion of the EVs depends on their capacity for energy storage. The second technology, HEV, has been developed to overcome the limitations of FEV with the aim to to extend the range capability. This technology consists of replacing a discharged battery by an alternative engine for propelling the EV and recharge its discharged battery. With this technology, the precipitation for searching an adequate charging station is not required. In the third technology, PHEV, the EVs have on-board engines that may be used to charge their batteries by regenerative breaking of motors or another electric source. An energy management system for Opel is developed in [1]. It consists in regulating the interaction between the electric motor, generator, gasoline engine and the battery.

When we discuss about EV charging process, it is also necessary to know about characteristics of charging stations and charging modes. For EV drivers there are two possibilities for charge their EVs. The first one is the use of an adequate electrical outlet installed at home overnight in a garage, and the second one is the use of public charging stations. Furthermore, slow or fast charging can be used according to the wishes and constraints of drivers. For example, slow charging equipment can be installed at home and public stations can provide the possibility for fast charging for EVs. It is worth noting that for slow and fast charging, there are three power levels. Level 1 is mainly used for charging at home. Power levels 2 and 3 are used in public charging stations.

Usually chargers are divided into two types: on-board and off-board with unidirectional or bidirectional power flow as detailed in [19] and [20]. In electricity flow when using AC/DC (Alternating Current/Direct Current) then this kind of charger which is connected with the Grid is off-board. Otherwise DC/DC converter, connected with the battery, is on-board charger.

4. PROBLEM STATEMENT AND MODELING APPROACH

In this section, we present the modeling approach that we adopt in this paper for describing the behavior of the main components of the system. More precisely, we represent the behavior of EV with various states (under charging, fully charged, under driving), an intermediate collaborative platform (denoted CPL) between EVs and charging stations (see [15] for more details) and finally the charging station with one or several charging points. The relationship and connection between these components are represented. The system is represented by a TEG model, and then described mathematically by a (max, +)-linear system. Based on the obtained (max, +) model, we show how to act on the charging time, or also charging rate (SoC of the battery), according to the arrival frequency of charging requests and concrete needs of energy for EVs in order to serve a maximum number of requests while avoiding a long waiting of vehicles. It is worth noting that the arrival frequency of charging demands is defined according to a given distribution (uniform, random, etc.). In our case, we consider a random arrival of charging requests. This will be further explained hereafter.

To prove the feasibility of the proposed approach, first we consider many charging requests addressed to a charging station with only one charging point via the CPL. Thereafter, we study the evolution of charging process (charging rate, charging time, waiting time for charging) by increasing the number of charging points or also the number of charging stations. In this study we suppose that all charging requests are addressed to charging stations with random time slots (arrival frequencies) and all charging points provide the same charging service (especially the same power).

4.1. MODELING WITH TEG

In order to adequately satisfy all received charging demands and accomplish the charging process successfully, we represent its behavior by a graphic-based model using the TEG formalism. As mentioned already previously, in this model, transitions of the model represents the events and their firings represent the occurrence of these events. A TEG is a subclass of Petri nets where each place has exactly one upstream (or incoming) transition and one downstream (or outgoing) transition. The weight on each arc equals to 1 by default. TEG is well known to be rather adapted to problems with synchronization and parallelism phenomena, and then which suppose the absence of conflicts and resources sharing. The dynamic of TEG is governed by a set of tokens that participate to the firing of transitions and change the system states. More details about this formalism can be found in [2], [5] and [10].

In a formal way, a PN is a 5-tuple $PN = (P, T, A, W, M_0)$ where:

• $P = \{p_1, \ldots, p_n\}$ is a finite set of places (represented by circles);

• $T = \{T_1, \ldots, T_n\}$ is a finite set of transitions (line segments);

• $A \subseteq (P \times T) \cup (T \times P)$ is a finite set of arcs;

• $W = A \rightarrow \{1,2, \ldots\}$ is the weight function associated with arcs (equal to 1 for all arcs of TEG);

• $M_0 = P \rightarrow \{0,1,2, \ldots\}$ is the initial marking of the graph.

These definitions are also true for TEG. In addition, when times are associated with certain places of the TEG the model is called P-Timed Event Graph, called also TEG in the rest of this paper. Also, when the sojourn times of tokens in certain places are given as time intervals, the graphic model is named P-Timed Event Graph (PTEG). More details about these varieties of PN are given in [5]. Without any ambiguity, when the time (fixed time or time interval) is considered, the graphic model is called TEG. In our study we suppose that all transitions are immediate transitions (transitions without firing delays).

The figure 1 presents the EG model of the charging process. It consists of three main parts as illustrated in the table 2:

• EV part with its access conditions to the CPL;

• Collaborative platform (CPL);

• Charging station with available charging points.

The EG model dynamic evolves according to the occurrence of different events and states of the process following a chronological order. For each charging request, the EV connects to the CPL (firing of *Conn-Ch-Serv*). The firing of the transition *Rec-Req-EV* means that the request is received by the CPL.

Thereafter, the CPL connects to the charging station. If this last is free (presence of a token in the place *Avail-CS*) then the charging point can be reserved for the EV asking to be charged. In this EG we consider only one charging point which is represented with the only token presented in the place *Avail-CS*. The number of charging points will be increased in the second part of this study. Next step is the traveling of EV for reaching the charging station. When the EV arrives at charging station, it can begin

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charging operation. The charging time (or charging rate) of this EV depends on the queue of EVs which wait for charging (this time will be introduced and evaluated with TEG and (max, +)-equations). The firing of the transitions *Beg-Charg* means that the EV starts its charging operation. In our study we consider that charging rate for this EV depends on the arrival date of the next charging request. We adopt this policy to avoid long waiting times of EVs for charging. At the end of charging operation, the charging point becomes available again and a notification is sent to the CPL (firing of *Notif-CPL*) in order to update the CPL data base. All these steps are represented and included in the EG model of figure 1. In table 2 we present all EG model nodes (places and transitions). Each place or transition has own short name (designation) and its significance.

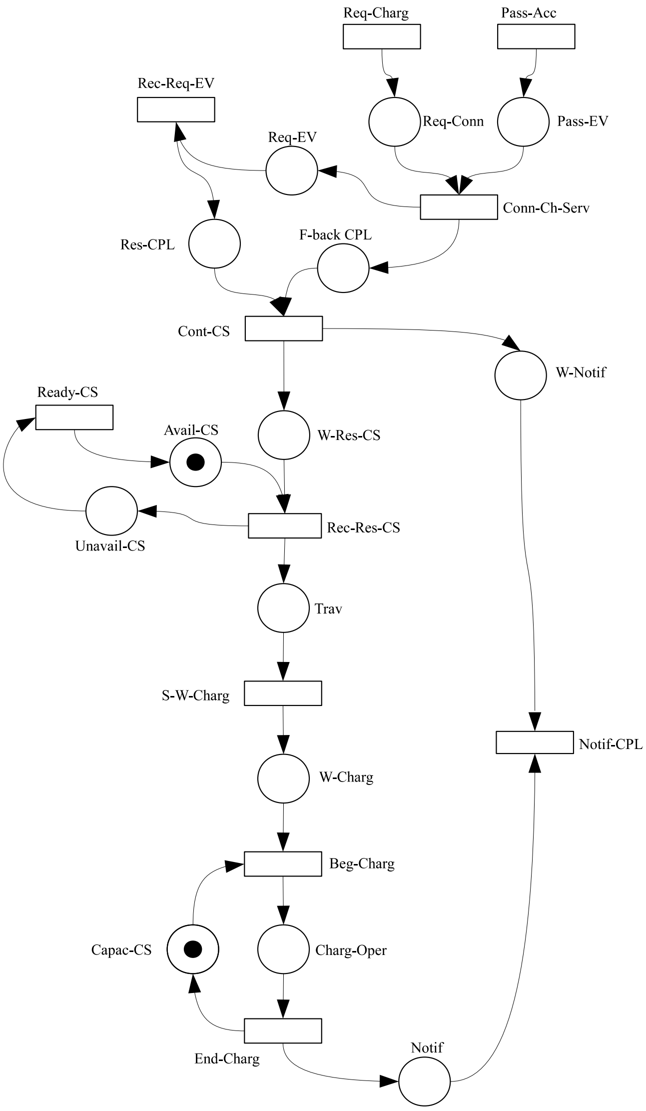

Figure 1. Event Graph model of the charging process.

The standard qualitative properties of the studied charging process are analyzed and verified by the execution of the EG model (figure 1) using the VisualObjectNet++ software. The proposed model is used for verifying the process feasibility and other properties such as sojourn times of tokens in each place, transition firings, reachable states, etc.. These properties are known in Petri net language as reachability, boundedness, deadlock-free.

Table 1. Legend of figure 1.

<table>
<tr><th>Entity</th><th>Node</th><th>Designation</th><th>Significance</th></tr>
<tr><td rowspan="12">EV</td><td rowspan="7">Places</td><td>- Req-Conn</td><td>- Charging request and connect to the CPL</td></tr>
<tr><td>- Pass-EV</td><td>- Driver password</td></tr>
<tr><td>- Req-EV</td><td>- EV request sent to the CPL</td></tr>
<tr><td>- F-back CPL</td><td>- Waiting the CPL feedback</td></tr>
<tr><td>- W-Notif</td><td>- Waiting notification to update the CPL DB</td></tr>
<tr><td>- Trav</td><td>- Traveling to the charging station</td></tr>
<tr><td>- W-Charg</td><td>- Waiting for charging</td></tr>
<tr><td rowspan="5">Transitions</td><td>- Req-Charg</td><td>- Request for charging</td></tr>
<tr><td>- Pass-Acc</td><td>- Password to access to the CPL</td></tr>
<tr><td>- Conn-Ch-Ser</td><td>- Connect and send a request to the CPL</td></tr>
<tr><td>- Cont-CS</td><td>- Sending EV request to a charging station</td></tr>
<tr><td>- S-W-Charg</td><td>- Waiting the availability of a CP</td></tr>
<tr><td rowspan="4">CPL</td><td rowspan="2">Places</td><td>- Res-CPL</td><td>- Response from the CPL</td></tr>
<tr><td>- W-Res-CS</td><td>- Waiting response from CS</td></tr>
<tr><td rowspan="2">Transitions</td><td>- Rec-Req-EV</td><td>- Receiving request from EV</td></tr>
<tr><td>- Rec-Res-CS</td><td>- Receiving response from a charging station</td></tr>
<tr><td rowspan="9">CS</td><td rowspan="5">Places</td><td>- Unavail-CS</td><td>- Unavailability of charging station</td></tr>
<tr><td>- Avail-CS</td><td>- Availability of the CS</td></tr>
<tr><td>- Capac-CS</td><td>- Capacity of the CS</td></tr>
<tr><td>- Charg-Oper</td><td>- Charging operation</td></tr>
<tr><td>- Notif</td><td>- Notification of the charging operation end</td></tr>
<tr><td rowspan="4">Transitions</td><td>- Ready-CS</td><td>- A charging station is ready to charge an EV</td></tr>
<tr><td>- Beg-Charg</td><td>- Beginning of charging operation</td></tr>
<tr><td>- End-Charg</td><td>- Ending of charging operation</td></tr>
<tr><td>- Notif-CPL</td><td>- Notifying CPL (update the CPL DB)</td></tr>
</table>

The next step of the study is the evaluation and analysis of other process performances such as occurrence time of each state (e.g. waiting, charging, notifying), process improvement, etc.. The process behavior is then described, by introducing time parameter on the graphic model and translating the EG model to TEG, using a representation state in (max, +) algebra.

Before representing the behavior of the EG model of figure 1 with mathematical equations, let us recall some basic elements of (max, +) algebra. More details about this algebra is given in [17].

4.2. BASIC ELEMENTS OF (MAX, +) ALGEBRA

The (max, +) algebra is a linear mathematical algebra endowed with several properties and assertions like the conventional algebra. In this algebra there are two main operators $\oplus$ and $\otimes$ named respectively maximum and addition. The set $\mathbb{R}_\varepsilon \overset{def}{=} \mathbb{R} \cup \{-\infty\}$ endowed with these two operators is called a dioid (i.e. $(\mathbb{R}_\varepsilon, \oplus, \otimes)$). For all $x, y \in \mathbb{R}_\varepsilon$,

$$x \oplus y = \max(x, y) \quad and \quad x \otimes y = x + y$$

Usually we call $\oplus$ the (max, +) addition, and $\otimes$ the (max, +) multiplication. The zero element for $\oplus$ is $\varepsilon \overset{def}{=} -\infty$. We can write: $\forall \mathrm{a} \in \mathbb{R}_\varepsilon, \mathrm{a} \oplus \varepsilon = \mathrm{a} = \varepsilon \oplus \mathrm{a}$. The neutral element of $\otimes$ is $e \overset{def}{=} 0$. That means: $\forall \mathrm{a} \in \mathbb{R}_\varepsilon, \mathrm{a} \otimes e = \mathrm{a} = e \otimes \mathrm{a}$. The element ε is called absorbing element for $\otimes$, $\forall \mathrm{a} \in \mathbb{R}_\varepsilon, \mathrm{a} \otimes \varepsilon = \varepsilon = \varepsilon \otimes \mathrm{a}$. Let $\mathrm{r} \in \mathbb{R}$ the r^{th} (max, +) – algebraic power of $\mathrm{x} \in \mathbb{R}_\varepsilon$ is denoted by $x^{\otimes^r}$ and corresponds to $r.x$ (with "." is the multiplication in conventional algebra). For $\mathrm{x} \in \mathbb{R}_\varepsilon$ then $x^{\otimes^0} = 0$ and the opposite element of x is $x^{\otimes^{-1}} = -x$. There is no opposite element for ε. If $r > 0$ then $\varepsilon^{\otimes^r} = \varepsilon$, else if $r < 0$ then $\varepsilon^{\otimes^r}$ is not defined.

Like for the conventional algebra, the matrix calculation in the (max, +) algebra is also possible and enables to solve infinity of problems. The basic (max, +) algebraic operations are extended to matrices as follows.

$\forall \mathrm{A}, \mathrm{B} \in \mathbb{R}_\varepsilon^{\mathrm{m}\times\mathrm{n}}$ and $\mathrm{C} \in \mathbb{R}_\varepsilon^{\mathrm{n}\times\mathrm{p}}$ (where $\mathbb{R}_\varepsilon^{\mathrm{m}\times\mathrm{n}}$ is a dioid of matrices with m lines and n columns. The elements of these matrices are scalars in $\mathbb{R}_\varepsilon$), we can write for all i, j:

$$(\mathrm{A} \oplus \mathrm{B})_{\mathrm{ij}} = \mathrm{a}_{\mathrm{ij}} \oplus \mathrm{b}_{\mathrm{ij}} = \max(\mathrm{a}_{\mathrm{ij}}, \mathrm{b}_{\mathrm{ij}})$$

$$(\mathrm{A} \otimes \mathrm{C})_{\mathrm{ij}} = \oplus_{\mathrm{k}=1}^{\mathrm{n}} \mathrm{a}_{\mathrm{ik}} \otimes \mathrm{c}_{\mathrm{kj}} = \max_{\mathrm{k}}(\mathrm{a}_{\mathrm{ik}} + \mathrm{c}_{\mathrm{kj}})$$

The matrix $\varepsilon_{\mathrm{m}\times\mathrm{n}}$ is the set of $\mathrm{m} \times \mathrm{n}$ (max,+) algebraic zero matrices with $(\varepsilon_{\mathrm{m}\times\mathrm{n}})_{\mathrm{ij}} = \varepsilon$ for all i, j.

The matrix E_n is the $\mathrm{n} \times \mathrm{n}$ (max,+) algebraic identity matrix with:

$(\mathrm{E}_\mathrm{n})_{\mathrm{ii}} = \mathrm{e}$ for all i and $(\mathrm{E}_\mathrm{n})_{\mathrm{ij}} = \varepsilon$ for all i, j with $i \neq j$. The (max, +) – algebraic matrix power of $\mathrm{A} \in \mathbb{R}_\varepsilon^{\mathrm{n}\times\mathrm{n}}$ is defined as follows: $\mathrm{A}^{\otimes^0} = \mathrm{E}_\mathrm{n}$ and $\mathrm{A}^{\otimes^k} = \mathrm{A} \otimes \mathrm{A}^{\otimes^{k-1}}$ for $k = 1,2, \dots$

The Kleene star of a matrix A is given by $A^* = E_n \oplus A \oplus A^{\otimes^2} \oplus$ .... We show later, how this matrix will be used to evaluate different states of the process whose behavior is expressed with a (max, +) implicit equation such as the equation (4.1) given hereafter. Also reminder the conditions of the existence of the Kleene star of a given matrix.

It is well known that the dynamic behavior of a TEG can be expressed by a system of linear inequalities in the (max, +) algebra [16]. To do so, we introduce the time parameter on the EG of figure 1 and we associate with each transition x_i of the TEG a dater, x_i(k) which corresponds to the date of its k^{th} firing. A dater is key element for evaluating the execution time (beginning and end) of each task of the process.

4.3. (MAX, +) – STATE REPRESENTATION

In this section, we show how (max, +) algebra will be used for modeling, analysis and performance evaluation of the charging process. We show later that the behavior of this process can be described by the following (max, +)-state model: $\forall k \geq 2$,

$$\begin{cases} X(k) = A_0 \otimes X(k) \oplus A_1 \otimes X(k-1) \oplus B \otimes U(k) \\ Y(k) = C \otimes X(k) \end{cases} \tag{4.1}$$

With:

• k is the k^{th} charging request sent by an EV;

• $U(k)$ is the arrival time of the k^{th} request. It is called input vector whose components correspond to the input transitions of the TEG model;

• $X(k)$ is the state vector containing the daters of all process operations (connection to the CPL, waiting, charging, notifying, updating, ...) for the k^{th} request. This vector is called state vector whose components are state variables associated with the internal transitions of the TEG model;

• A_0, A_1, B and C are the characteristic matrices of the process. These matrices contain the required times to perform all tasks of the process from connection to the CPL by an EV until receiving a notification. As known in the literature all given data of the system are expressed in these matrices;

• $Y(k)$ is the notification time of ending charging process for the k^{th} request. It represents the output vector whose components correspond to the output transition of the TEG model;

• The first equation enables to evaluate the system evolution and computes its different states. The three terms on the right are given such that the two first terms $A_0 \otimes X(k)$ and $A_1 \otimes X(k-1)$ represent the impact of the internal state of the process on its evolution, and the second one $B \otimes U(k)$ models the impact of the process input on its evolution.

• The second equation enables to compute the system output and the term $C \otimes X(k)$ represents the impact of the system internal state on its output.

In order to translate (describe) the obtained graphical model (figure 1) into mathematical equations in (max, +)-algebra as the system (4.1), we first associate a variable to each transition as shown in figure 2. Thus, we associate input variables (denoted by u_1 and u_2 with input transitions and state variables $(x_1, x_2, \dots, x_{11})$ with internal transitions, and finally we associate output variables (denoted by y_1 and y_2) with the output transitions. As mentioned previously, we also assign time intervals to certain places. A time interval associated with a given place means that the sojourn time of a token in this place varies between a lower and an upper bound. All places with time intervals represent the process components, e.g. CPL, for which the responses are not often immediate and require a time for answering a query. Time intervals can also be assigned to places wherein the tokens wait for responses, e.g. an EV waiting to be charged. These time intervals will be the key elements in the performances improvement of the process. Fixed times which represent the necessary time to accomplish a task of the process is assigned with other places of the model.

Taking into account these explanations and introducing the new variables, the fixed times and time intervals the EG model of 1 becomes a TEG model as given in figure 2.

The second stage of the analytic modeling is to label each model variable x_i $(1 \leq i \leq 8)$ by $x_i(k)$ for all $k \geq 1$, called "*dater*", which denotes the time of the k^{th} firing of x_i. Similarly we define the daters $u_i(k)$ $(1 \leq i \leq 2)$ and $y(k)$.

Using all daters and temporizations (time intervals and fixed times), we give hereafter a system of (max, +)-linear equations that represent the process.

Let consider the following parameters:

• CT_{max}: Required time for a full charging $(SoC = 100\%)$ of the EV battery;

• CT_{min}: Required time for a minimum acceptable charging rate;

• $TT(k)$: Trip Time from the location of k^{th} EV to the charging station.

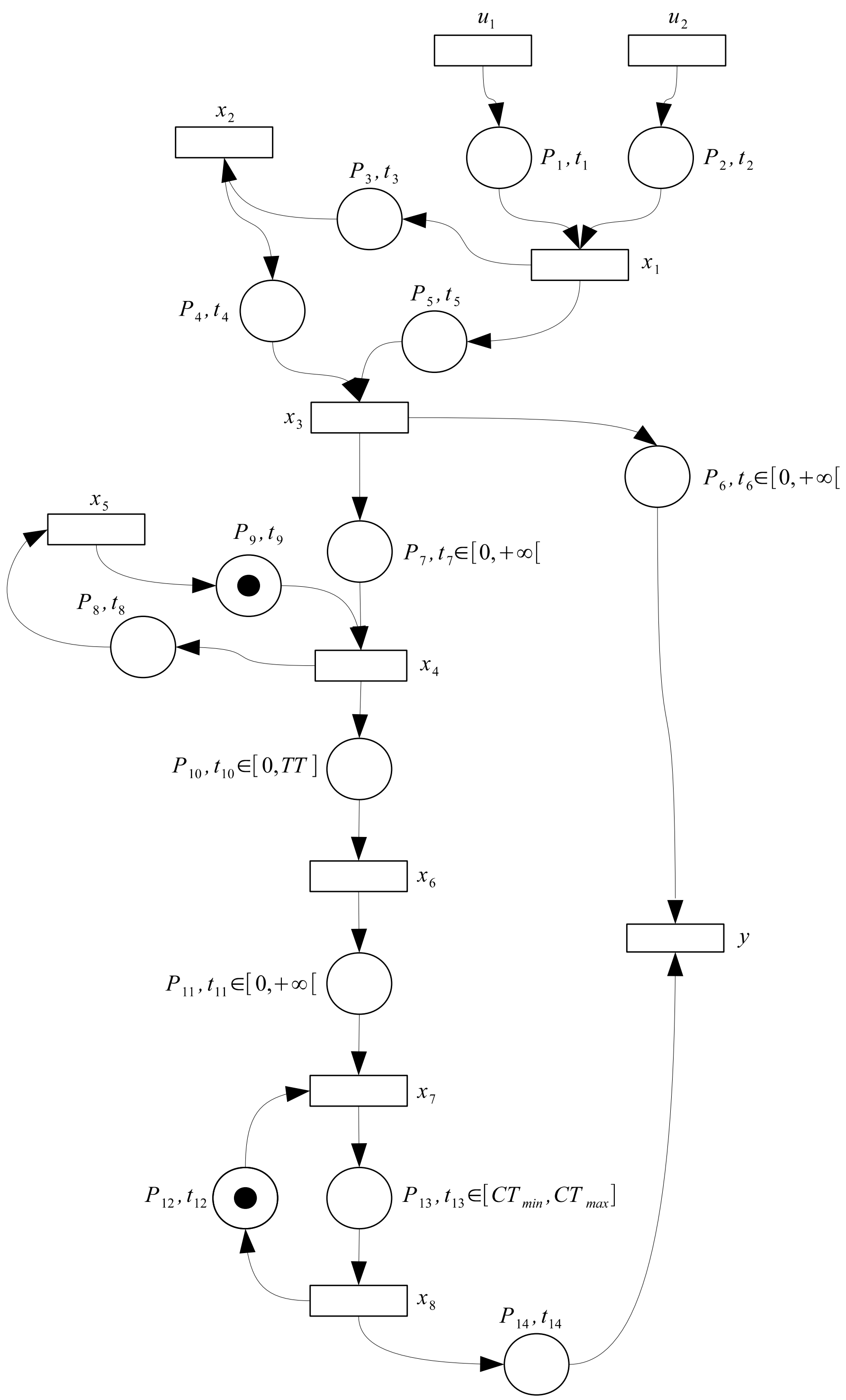

Figure 2. TEG model of the charging process labeled with variables and temporizations.

We denote by P_i, representing a system state, the output place (resp. input place) of the transition x_i (resp. of x_j). These transitions represent respectively the beginning and the ending of a process task (represented by P_i). The required time to accomplish this task is denoted t_i and associated with the place P_i. The temporization t_i corresponds to the sojourn time of tokens in the place P_i. In our model, these temporizations represent needed times to achieve tasks by involved services in the process.

In order to show how to translate a TEG behavior into a mathematical equation, first we express the dater $x_1(k)$ as given by the equation (4.2). $\forall k \geq 1$,

$$\begin{cases} x_1(k) = Max\ (t_1 + u_1(k), t_2 + u_2(k)) \\ \quad = t_1 \otimes u_1(k) \oplus t_2 \otimes u_2(k) \end{cases} \tag{4.2}$$

The equation (4.2) means that the k^{th} firing of x_1 can occurs only after t_1 time unit after the firing of u_1 and after t_2 time unit after the firing of u_2.

The (max, +) equations representing the system are given by the model (4.3). $\forall k \geq 2$

$$\begin{cases} x_1(k) = t_1 \otimes u_1(k) \oplus t_2 \otimes u_2(k) \\ x_2(k) = t_3 \otimes x_1(k) \\ x_3(k) = t_4 \otimes x_1(k) \oplus t_5 \otimes x_2(k) \\ x_4(k) = t_7 \otimes x_3(k) \oplus t_9 \otimes x_5(k-1) \\ x_5(k) = t_8 \otimes x_4(k) \\ x_6(k) = t_{10} \otimes x_4(k) \\ x_7(k) = t_{11} \otimes x_6(k) \oplus t_{12} \otimes x_8(k-1) \\ x_8(k) = t_{13} \otimes x_7(k) \\ y(k) = t_6 \otimes x_3(k) \oplus t_{14} \otimes x_8(k) \end{cases} \tag{4.3}$$

with: $t_6 \in [0, \ +\infty[$, $t_7 \in [0, \ +\infty[$, $t_{10} \in [0,\ TT]$, $t_{11} \in [0, \ +\infty[$, $t_{13} \in [CT_{min},\ CT_{max}]$. The other temporizations of the system are supposed to be fixed.

The equations of the system (4.3) will be written as a 1^{st} order recurrent matrix equation in order to facilitate its resolution. In doing so, we define the following vectors:

- Input vector $U = [u_1,\ u_2]^t$;
- State vector $X = [x_1,\ x_2,\ x_3,\ x_4,\ x_5,\ x_6,\ x_7,\ x_8]^t$;
- Output vector $Y = y$.

By using these vectors, the equations of the system (4.3) can be written as the standard equation (4.1), where $A_0 \in \mathbb{R}_{max}^{8\times8}$, $A_1 \in \mathbb{R}_{max}^{8\times8}$, $B \in \mathbb{R}_{max}^{8\times2}$, $C \in \mathbb{R}_{max}^{1\times8}$ are the characteristic matrices of the model (containing all system data).

4.4. (MAX, +) – STATE RESOLUTION

In order to solve the implicit equation given by the system (4.1), we proceed as follows: we replace in the first equation of

(4.1), successively, $X(k)$ by its expression to obtain the smallest solution of the first equation of (4.1). This allows to express the equation according to the Kleene star A_0^* of the matrix A_0. If such matrix is defined, then the implicit system has a realizable solution, otherwise the solution doesn't exists and then the feasibility of the process can not be proved. Thus the first equation of the system (4.1) can be written as follows: $\forall k \geq 2$,

$$\begin{cases} X(k) = A_0X(k) \oplus A_1X(k-1) \oplus BU(k) \\ \quad = A_0[A_0X(k) \oplus A_1X(k-1) \oplus BU(k)] \\ \quad \oplus A_1X(k-1) \oplus BU(k) \\ \quad = A_0^2X(k) \oplus A_0A_1X(k-1) \oplus A_0BU(k) \\ \quad \oplus A_1X(k-1) \oplus BU(k) \\ \quad = A_0^2X(k) \oplus [A_0 \oplus Id]A_1X(k-1) \\ \quad \oplus [A_0 \oplus Id]U(k) \\ \quad = \cdots \\ \quad = A_0^nX(k) \oplus \bigoplus_{i=0}^{n-1} A_0^i[A_1X(k-1) \\ \quad \oplus BU(k)] \\ \quad = \bigoplus_{i=0}^{n-1} A_0^iA_1X(k-1) \oplus \bigoplus_{i=0}^{n-1} A_0^iBU(k) \\ \quad = (\bigoplus_{i=0}^{n-1} A_0^i)A_1X(k-1) \oplus (\bigoplus_{i=0}^{n-1} A_0^i) \\ \quad BU(k) \\ \quad = A_0^*A_1X(k-1) \oplus A_0^*BU(k) \end{cases} \tag{4.4}$$

where A_0^* is defined by: $A_0^* = \bigoplus_{i=0}^{+\infty} A_0^i$. As mentioned previously about the calculation of the Kleene star A_0^*, A_0^n for $n \geq 9$ (A_0 is an (8x8) matrix), does not contribute to the sum of A_0^*. In order terms, $\forall n \geq 9$, $A_0^n = \varepsilon$. So, $A_0^nX(k) = \varepsilon$, $\forall n \geq 9$ and $\forall k \geq 2$. Let us recall that the existence of Kleene star A_0^* leads to a feasible solution of the equation (4.1). The matrix Id introduced in the system (4.5) is the identity matrix (E_n with $n = 8$) as given in the section of basic elements of (max, +) algebra.

The evaluation of the system will be done knowing that the numerical values of the system input $U(k)$, for all k, and the system initial state $X(1)$ are given. The solution of the system (4.1) is then given by: $\forall k \geq 2$,

$$\begin{cases} X(k) = A_0^* A_1 X(k-1) \oplus A_0^* B U(k) \\ \quad = (A_0^* A_1)^{k-1} X(1) \oplus \bigoplus_{i=0}^{k-2} (A_0^* A_1)^i \\ \quad (A_0^* B) U(k-i) \\ Y(k) = C X(k) \\ \quad = C((A_0^* A_1)^{k-1} X(1) \oplus \bigoplus_{i=0}^{k-2} (A_0^* A_1)^i \\ \quad (A_0^* B) U(k-i)) \end{cases} \quad (4.5)$$

4.5. NUMERICAL EXAMPLE: ANALYSIS AND EVALUATION

For the evaluation study, we will assign numerical values to various parameters as shown in Table 2. These various numerical values are defined as follows. Each system operation can be done within a given time interval $[a,\ b]$, where the lower bound a is the required minimum time to perform the task and the upper bound b is the maximum time not to exceed for executing the task. The values of t_{10} and t_{12} are given within time intervals (as given in Table 2) according to some criteria, such as: the charging operation, availability of a charging point to perform a charging operation. Other parameters t_6, t_7 and t_{11} represent the waiting for receiving response and then to perform next charging operation. These waiting times vary from "0" which means that the reply for charging request is immediate to "$+\infty$" which means that the response will never be received. In most cases, the waiting time is defined and bounded. In addition, we carry out a feasibility study and performances improvement of the process. For a concrete application, these timing parameters may be changed slightly but the principle remains the same.

Table 2. Numerical values of system parameters.

Times	t_1	t_2	t_3	t_4	t_5	t_6	t_7
Num. values	0	0	0	0	0	∈[0, +∞[	∈[0, +∞[
	t_8	t_9	t_{10}	t_{11}	t_{12}	t_{13}	t_{14}
	0	0	∈[0, TT[	∈[0, +∞[	0	$\in[CT_{min}, CT_{max}[$	0

We assume that the arrival dates of charging requests follow a stochastic low (not uniform) and known $apriori$. These arrival dates are given by the daters $u_1(k)$, for all $k \geq 1$. The performed simulations and obtained results are based on these arrival dates. Furthermore, we assume that the maximal and minimal charging times are: $CT_{max} = 40\ t.u$, $CT_{min} = 20\ t.u$ (with t.u means time unit). As said previously, we predict charging rate and time for each charging request according to the arrival frequency of all EV request on one hand, and the travel time ($TT(k)$) of each k^{th} EV from its location to the charging station on the other hand. Charging times $t_1(k)$ and charging rates are calculated according to the flowchart of figure 3. In this algorithm, $\Delta(k)$ is defined as follows. For all $k \geq 2$, $\Delta(k)$ is expressed according to the time slot between two consecutive arrivals of k^{th} and $(k-1)^{th}$ charging demands including the theoretical waiting time denoted $wt(k)$ of the k^{th} demand before freeing the charging point by the $(k-1)^{th}$ EV. The travel time from EV location till the charging station is included in the expression of $\Delta(k)$. The expression of this parameter is given by:

$$\forall k \geq 2,\ \Delta(k) = |v_1(k) - v_1'(k-1)| \qquad (4.6)$$

with: $v_1(k) = u_1(k) + TT(k), \quad v_1'(k-1) = v_1(k-1) + wt(k-1)$

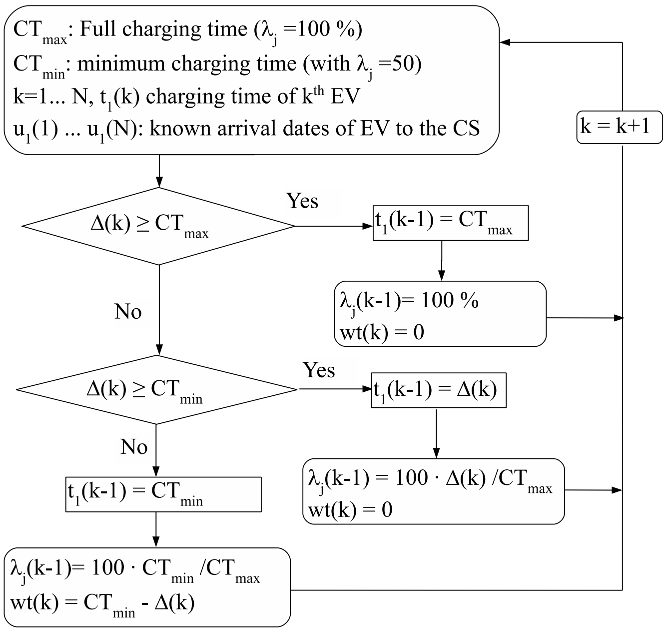

Figure 3. Flowchart of charging time and rate calculation.

Based on the proposed algorithm (figure 4), figure 6 presents a comparison between charging times and arrival of charging requests. This figure presents the evolution of each parameter versus time. As illustrated in this figure, charging time varies between 20 and 40 t.u according to the fixed charging bounds $CT_{min} = 20$, $CT_{max} = 40$. From these results we remark that the $(k-1)^{th}$ EV can be charged fully when the inter-arrival dates of $(k-1)^{th}$ and $(k)^{th}$ requests is superior to $\Delta(k)$. This means that the $(k-1)^{th}$ EV has enough time to be charged fully without any obligation to leave the station before being charged completely (λ = 100%). When the inter-arrival of two consecutive requests is too-short the charging time decreases to reach $CT_{min} = 20$ some times. In this case the first EV ($(k-1)^{th}$ EV) has to stop its charging and leave the charging point when the acceptable charging minimum rate is reached. Some times, the next EV ($(k)^{th}$ EV) has to wait till the EV under charging ($(k-1)^{th}$ EV) reaches the acceptable charging minimum rate.

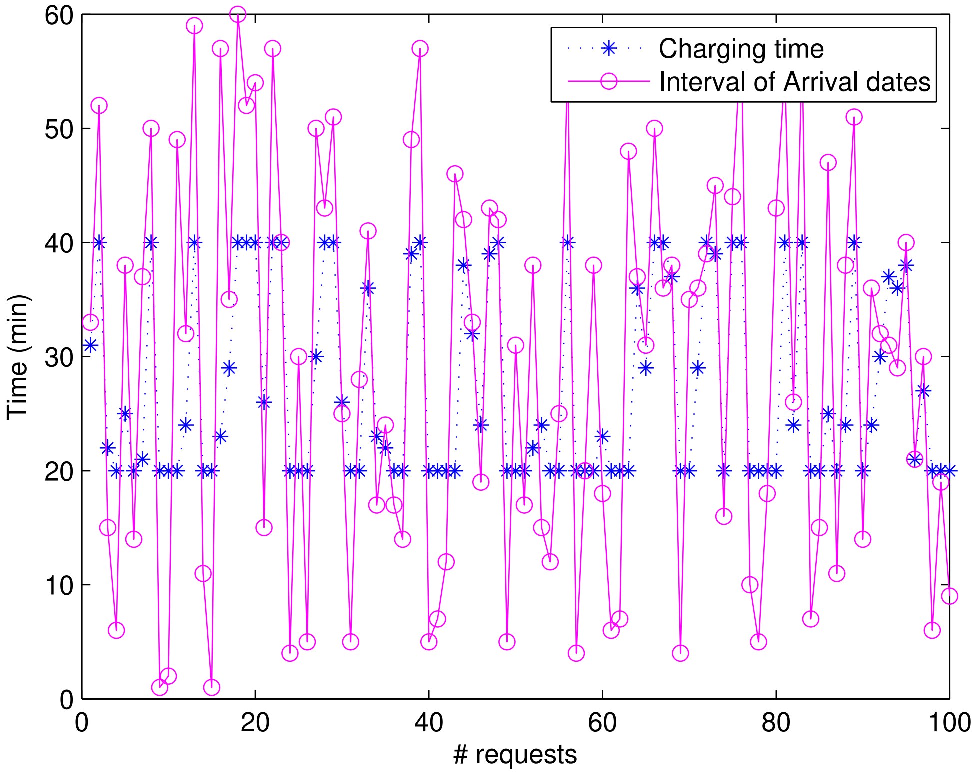

Figure 4. Comparison of the evolution of charging time with arrival dates of charging requests.

The main objective is to calculate the exact time of charging process for each EV. As depicted in figure 5, times of charging process for all EVs are not the same. In fact, as illustrated in figure 4, these charging times depend of the inter-arrival of consecutive requests. It depends also on the travel time from the EV location till the charging station. In figure 5, we limit the x-axis to only to 10 requests in order to show clearly the difference between the two curves (arrivals of charging requests and end of charging processes).

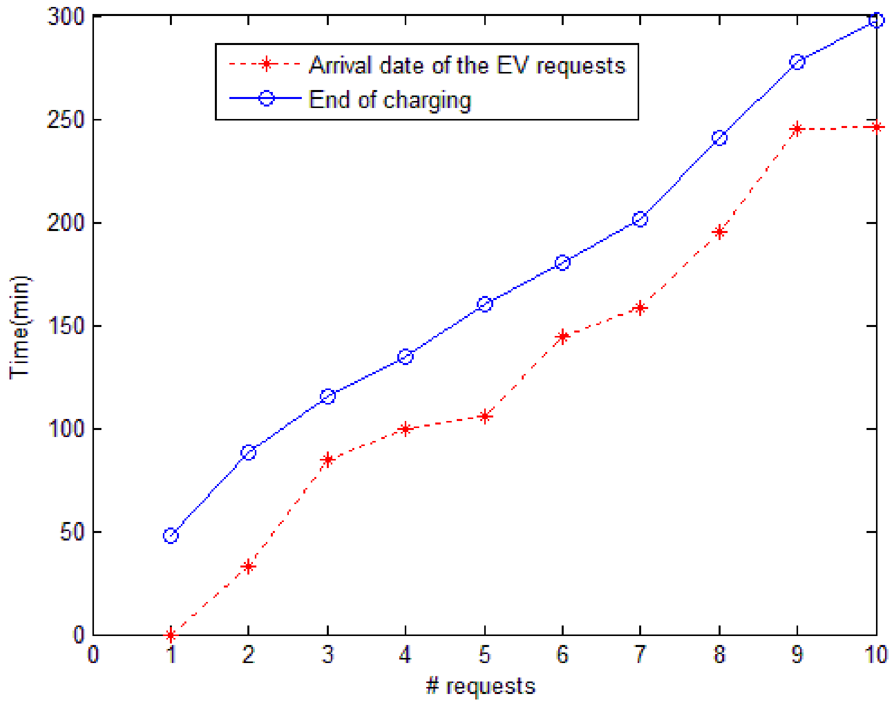

Figure 5. Difference between arrival dates of the requests and the end of charging.

As mentioned previously, we aim to charge EVs fully as more as possible. Using only one charging point and according to arrival dates and travel times of EVs to reach charging station, we can define exact number of fully charged EVs (see figure 6). In this case, the number of EVs fully charged is very low compared with the number of received charging requests.

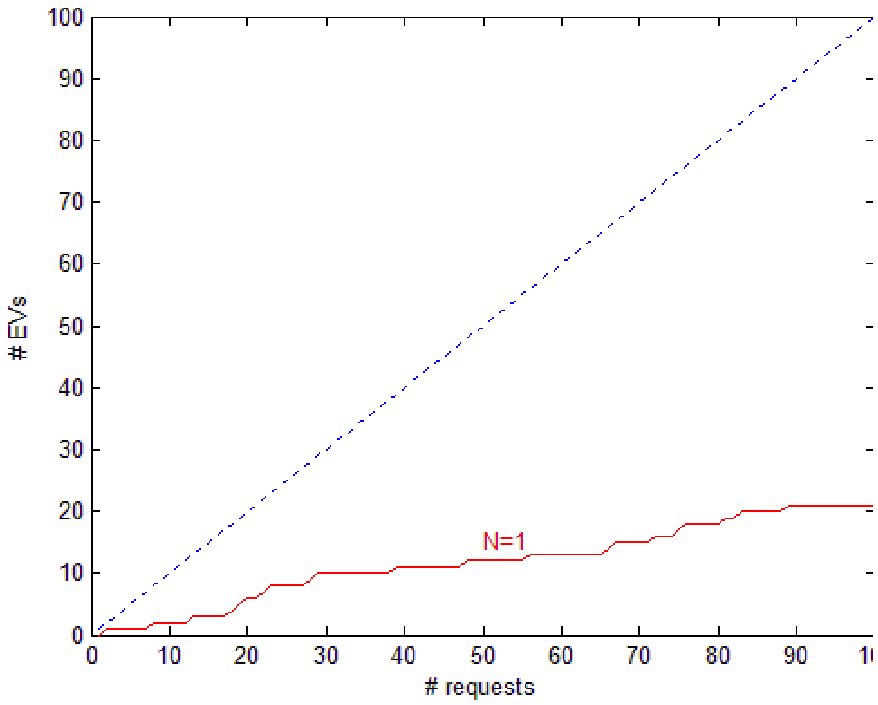

Figure 6. Number of EVs fully charged vs number of charging requests using one CP.

The next step is to improve the obtained results by increasing the number of EVs fully charged. We show, using the simulation of the proposed (max, +) model, how this number evolves according the number of used charging points.

5. IMPROVEMENTS: SIZING RESULTS

The objective of this improvement study is to increase charging rate of each EV (ideally reaching a SoC of 100 %). For doing so, we show how the increasing of the number of charging points (the minimum possible) participates to reach this goal. The figure 7 shows the rate of fully charged EVs according to the number of used charging points. Let note that for all obtained results in this case, we consider the same numerical values given in Table 2 and the same arrival dates of all charging requests.

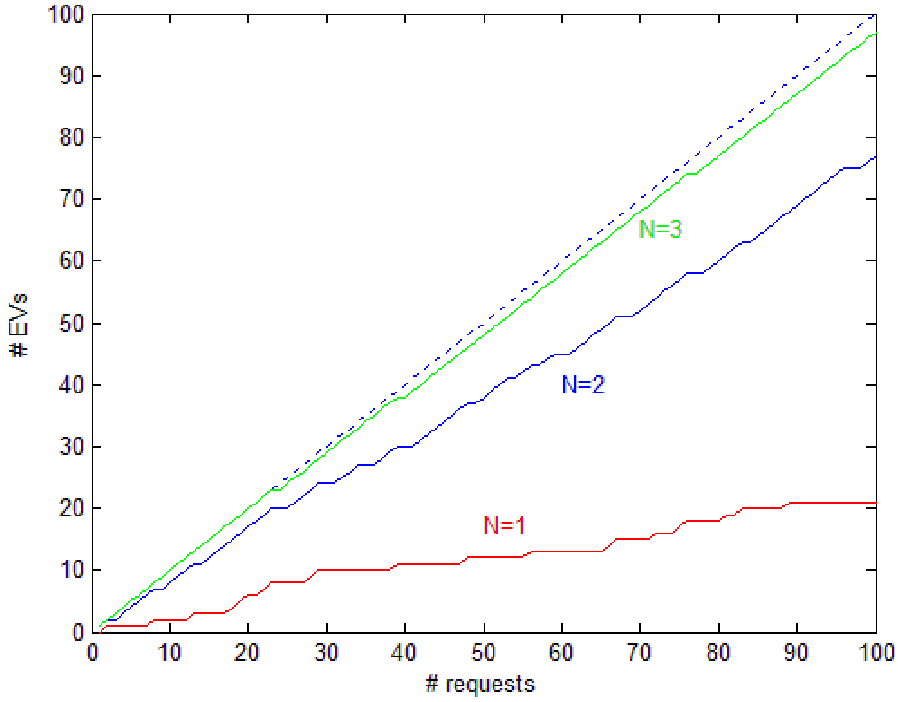

Figure 7. Number of EVs fully charged vs number of charging requests using 1, 2 and 3 CPs.

The simulation results show that, when only one charging point is used 21% of EVs are fully charged. Using two charging points we remark that 77% of all EVs can be fully charged . Finally 97% of EVs can be fully charged by using three charging points (see figure 7). As a conclusion, with only three charging points within the charging station, the full charging of almost all EVs is ensured. Using four charging points, the obtained results, that we do not present in this paper, show that all EVs can be fully charged and the four CPs are not fully exploited.

6. CONCLUSIONS AND FUTURE WORK

In this paper, a charging process of electric vehicles was modeled, evaluated and improved using (max, +)-algebra. The process was first modeled using a dynamic timed event graph, and some appropriate properties such as feasibility, reachability, deadlock-free, synchronization, etc. of the process were evaluated through this graphical model. (Max, +)-equations describing the analytic behavior of the process are then derived from the TEG model. The required performance metrics are evaluated using these linear equations. Among these metrics we have evaluated the occurrence times of all process events and states, process evolution over time, feasibility of charging process. This is by proving the existence of a feasible solution for the implicit (max, +) equation through the existence of a determined form of Kleene star of the characteristic matrix of the (max, +) system.

A performance tuning algorithm was proposed on in order to improve the quality of service offered to EVs by increasing the charging rate. This tuning method allows to study the trade off between the number of charging requests and the number of charging points required to satisfy them, e.g., charge a maximum number of EVs using a minimum number of charging points. Furthermore, this study can be seen as a predictive charging policy to anticipate the assignment and the guidance of EV to charging stations. A numerical example was worked out and simulation results are reported and show the added value of the proposed predictive charging approach.

Throughout this study, we demonstrated, using a real-world application, how the proposed methodology can be used for validation of qualitative properties as well as issues of performance analysis, evaluation, and improvement.

In future work, we will extend this methodology to model and evaluate the performance of a complex and large distributed charging system. More precisely, the behavior of the proposed charging process will be modeled as a probabilistic/stochastic process. The proposed model will be extended while combining (max, +) algebra with stochastic processes to predict the charging process for EVs while considering the dynamic evolution of the system. In addition, real time issue will be considered and integrated in the developed model. In addition the correctness of the proposed approach will be further proved by performing some simulations and concrete experiments. The objective is to validate the obtained numerical results by these simulations and these experiments.

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