MATEMATIK ANALIZ FANINING SIRT INTEGRALLAR MAVZUSINI O'QITISHDA AXBOROT-KOMMUNIKATSION TEXNOLOGIYALARIDAN FOYDALANISH ASOSLARI

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ANNOTATSIYA

Ushbu maqolada birinchi tur sirt integrallarni hisoblash va sirt integrallarning ba'zi tadbiqlariga doir, ya'ni bir nechta jismlarning massasini topishga oid misollarning ishlanish usullari ko'rib chiqilgan va oliy ta'lim muassasalari talabalariga o'rgatishning qulay metodikasi tahlil qilingan.

Kalit so'zlar: integrallar, sirt integrallar, birinchi tur sirt integrallar, karrali integrallar, sirt integral tadbiqlari.

ABSTRACT

This article deals with the calculation of the first type of surface integrals and some applications of surface integrals, ie the development of examples of finding the mass of several bodies and analyzes a convenient method of teaching students in higher education.

Keywords: integrals, surface integrals, first type surface integrals, multiple integrals, surface integral applications.

KIRISH

O'zbekiston Respublikasi Prezidentining 07.05.2020 yildagi PQ-4708-sonli "Matematika sohasidagi ta'lim sifatini oshirish va ilmiy-tadqiqotlarni rivojlantirish chora tadbirlari to'g'risida"gi qarorining 1-ilovasidagi 3.1-bo'limining 14-"Matematika bakalavriat ta'lim yo'nalishlari bitiruvchilarining muayyan aniq sohalarda amaliy masalalarni yechish ko'nikmalarini rivojlantirish uchun ta'lim dasturlarini fanlar (yo'nalishlar)aro integrativ prinsip asosida ixtisoslashtirilgan tartibda ishlab chiqish va joriy etish." bandiga ko'ra bugungi kunga kelib, oliy ta'lim muassasalarida matematika fanini kasbga yo'naltirib o'qitish va hayotiy bog'liqlikda isbotlab o'qitish asosiy vazifalardan biri bo'lib qolmoqda [4]. Shularni inobatga olib ushbu maqolada asosan oliy ta'lim muassasalarining matematika darslari dasturiga kiritilgan sirt integrallar va ularning tadbiqlari mavzusini bir nechta misollardagi tadbiqlarini ko'rib o'tamiz.

Birinchi tur sirt integrallar va ularning tadbiqlarini misollardagi tahlilini ko'rib chiqishdan oldin sirt integrallarni hisoblash va sirt inegrallar yordamida massani topishdagi bir nechta qo'llash mumkin bo'lgan formulalari bilan tanishib olamiz va misollardagi tadbiqlarini ko'rib chiqamiz.

ADABIYOTLAR TAHLILI VA METODOLOGIYA

Faraz qilaylik, R^3 fazoda (S) sirt $z = z(x, y)$ tenglama bilan berilgan bo'lib, $z(x, y)$ funksiya chegaralangan (D) sohada uzluksiz va (D) da uzluksiz $z'_x(x, y)$, $z'_y(x, y)$ xususiy hosilalarga ega bo'lsin[1].

1-teorema. Agar $f(x, y, z)$ funksiya (S) sirtda berilgan va uzluksiz bo'lsa, u holda bu funksiyaning (S) sirt bo'yicha birinchi tur sirt integrali

$$\iint_{(S)} f(x, y, z)ds$$

mavjud va

$$\iint_{(S)} f(x, y, z)ds = \iint_{(D)} f(x, y, z(x, y))\sqrt{1+(z'_x)^2+(z'_y)^2}\,dxdy \quad (1)$$

bo'ladi.

Birinchi tur sirt integralining ba'zi tadbiqlari. Birinchi tur sirt integrallaridan sirtning yuzini, massani hisoblashda, og'irlik markazining koordinatalarini hamda inersiya momentlarini topishda ham foydalaniladi[1,2].

1. (S) sirtning yuzi

$$S = \iint_{(S)} ds \quad (2)$$

formula yordamida topiladi.

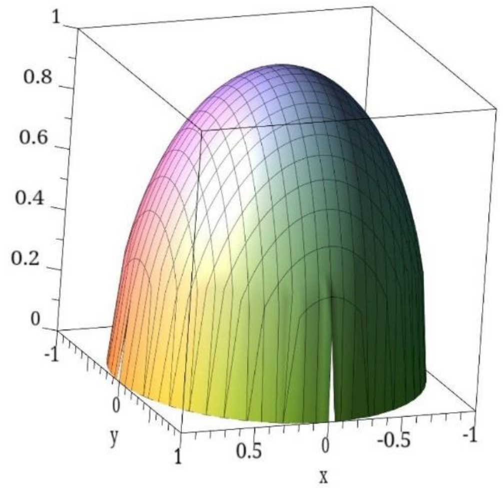
2. Agar (S) sirt bo'yicha zichligi $p(x, y, z)$ bo'lgan massa tarqatilgan bo'lsa, unda (S) sirtning massasi

$$m = \iint_{(S)} p(x, y, z)ds \quad (3)$$

bo'ladi.

1-misol. Ushbu $\iint_{(S)} xds$ integralni hisoblang, bunda (S) sirt quyidagi $z = \sqrt{1-x^2-y^2}$ yarim sferadan iborat[2].

Yechilishi.



1-rasm

Qaralayotgan S soha $z=\sqrt{1-x^2-y^2}$ tenglama bilan ifodalanadi(1-rasm)[5]. Bunda $z=z(x,y)$ funksiya $D=\{(x,y)\in R^2 : x^2+y^2\le 1\}$ da uzluksiz hamda uzluksiz xususiy hosilalarga ega:

$$z'_x=\frac{-x}{\sqrt{1-x^2-y^2}} \qquad z'_y=\frac{-y}{\sqrt{1-x^2-y^2}}$$

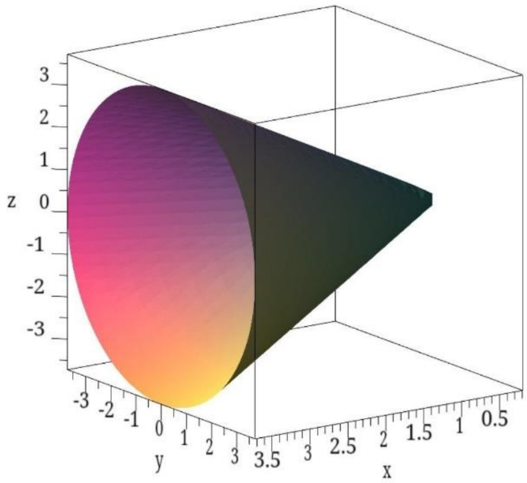
Bu D soha $z=\sqrt{1-x^2-y^2}$ sirtning XoY tekislikdagi proyeksiyasi. Quyidagi birinchi tur sirt integralni berilgan (1) formula yordamida hisoblaymiz:

$$S=\iint\limits_{(S)} x\,ds=\iint\limits_{(D)} x\sqrt{1+\left(\frac{y}{\sqrt{1-x^2-y^2}}\right)^2+\left(\frac{x}{\sqrt{1-x^2-y^2}}\right)^2}\,dxdy=\int_{-1}^{1}dy\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}}\frac{x}{\sqrt{1-x^2-y^2}}dx=$$

$$\int_{-1}^{1}\left(\int_{-\sqrt{1-y^2}}^{0}\frac{x}{\sqrt{1-x^2-y^2}}dx+\int_{0}^{\sqrt{1-y^2}}\frac{x}{\sqrt{1-x^2-y^2}}dx\right)dy=\int_{-1}^{1}-\sqrt{1-y^2}\,dy+\int_{-1}^{1}\sqrt{1-y^2}\,dy=0$$

Javob: 0

2-misol. Ushbu $\iint\limits_{(S)}(5x^2+3y^2+3z^2+4)ds$ integralni hisoblang, bunda (S) sirt quyidagi $x=\sqrt{y^2+z^2}$ tenglama bilan berilgan sirtning $x=0, x=2$ tekisliklar orasidagi qismi[3].



2-rasm

2-rasmda tasvirlangan[5] S soha $x = \sqrt{y^2 + z^2}$ tenglama bilan ifodalanadi. Bunda $x = x(y,z)$ funksiya $D = \{(y,z) \in R^2, y^2 + z^2 \le 4\}$ da uzluksiz va uzluksiz xususiy hosilalarga ega:

$$x_y' = \frac{y}{\sqrt{y^2 + z^2}} \qquad x_z' = \frac{z}{\sqrt{y^2 + z^2}}$$

Berilgan birinchi tur sirt integralni (1) formula yordamida hisoblaymiz:

$$S = \iint_{(S)} (5x^2 + 3y^2 + 3z^2 + 4)ds = \iint_{(D)} (5(y^2 + z^2) + 3y^2 + 3z^2 + 4)\sqrt{1 + \frac{y^2}{y^2 + z^2} + \frac{z^2}{y^2 + z^2}}dydz =$$
$$4\sqrt{2}\iint_{(D)} (2y^2 + 2z^2 + 1)dydz$$

Ikki karrali integralni hisoblash uchun quyidagicha shakl almashtiramiz:

$$\begin{cases} y = p\cos\alpha \\ z = p\sin\alpha \end{cases}, \quad \begin{matrix} 0 \le p \le 2 \\ 0 \le \alpha \le 2\pi \end{matrix} \qquad I = p$$

$$4\sqrt{2}\iint_{(D)} (2y^2 + 2z^2 + 1)dydz = 4\sqrt{2}\iint_{(D)} (2p^2 + 1)pdpd\alpha = 80\sqrt{2}\pi$$

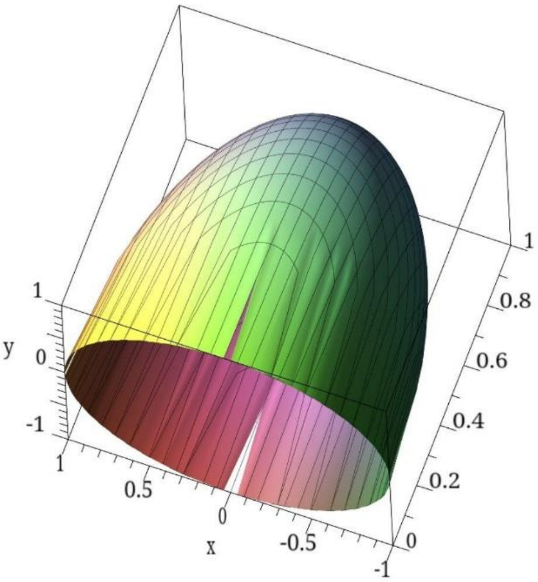
Javob: $80\sqrt{2}\pi$

3-misol. Zichligi $p(x,y,z) = x^2 + y^2 + z^2$ bo'lgan ushbu $z = \sqrt{r^2 - x^2 - y^2}$ yarimsferaning massani toping[3].

Yechilishi. Massani toppish formulasiga ko'ra

$$m = \iint\limits_{(S)} p(x,y,z)ds = \iint\limits_{(S)} (x^2 + y^2 + z^2)ds$$

Bu yerda (S) sirt $z = \sqrt{r^2 - x^2 - y^2}$ yarimsferadan iborat bo'lib, uning XoY tekislikdagi proyeksiyasi $D = \{(x,y) \in R^2 : x^2 + y^2 \le r^2\}$ dan iborat(3-rasm)[5].



3-rasm

(3) formuladan foydalanib, massani topamiz:

$$m = \iint\limits_{(S)} p(x,y,z)ds = \iint\limits_{(S)} (x^2 + y^2 + z^2)ds = \iint\limits_{(D)} (x^2 + y^2 + z^2)\sqrt{1 + (z_x')^2 + (z_y')^2}\,dxdy =$$

$$\iint\limits_{(D)} (x^2 + y^2 + r^2 - x^2 - y^2)\sqrt{1 + \frac{x^2}{r^2 - x^2 - y^2} + \frac{y^2}{r^2 - x^2 - y^2}}\,dxdy = r^3 \iint\limits_{(D)} \frac{1}{r^2 - x^2 - y^2}dxdy$$

Ikki karrali integralni ishlash uchun shakl almashtirishdan foydalanamiz:

$$\begin{cases} x = p\cos\alpha \\ y = p\sin\alpha \end{cases}, \quad \begin{matrix} 0 \le p \le 2 \\ 0 \le \alpha \le 2\pi \end{matrix} \quad I = p$$

$$m = r^3 \iint\limits_{(D)} \frac{1}{r^2 - x^2 - y^2}dxdy = r^3 \int_0^{2\pi} [\int_0^r \frac{pdp}{\sqrt{r^2 - p^2}}dp]d\alpha = 2\pi r^4$$

Javob: $2\pi r^4$

XULOSA

Sirt integrallarni hisoblash va sirt integrallarning ba'zi tadbiqlarini misollar yordamida o'rganish orqali matematika fanini boshqa fanlardagi tutgan o'rnini ham ko'rsatish mumkin. Masalan, ushbu maqolada ko'rib chiqilgan birinchi tur sirt integrallar yordamida jismlar massasini hisoblashga oid misollar nafaqat matematika fanida, balki fizika, kimyo va shu bilan birga texnikaning bir qator muammolarini hal qilishda ham uchrab turadi. Shulardan kelib chiqqan holda ushbu mavzu to'la yoritib berilishi orqali oliy ta'lim muassasalari talabalarining fanga bo'lgan qiziqishi, mantiqiy fikrlashi va muammoni kreativ yondashuv yo'li bilan hal qilish ko'nikmasini shakllantiriladi.

REFERENCES

1. T.Azlarov, H.Mansurov. "Matematik analiz" 1-qism, 2-qism. Toshkent "O'qituvchi"-1994.
2. A.Gaziyev, I.Isroilov, M.Yaxshiboyev. "Matematik analizdan misol va masalalar" 2-qism. Toshkent-2012.
3. A.Sadullayev, H.Mansurov, G.Hudoyberganov, A.Vorisov, R.G'ulomov. "Matematik analiz kursidan misol va masalalar to'plami" 1-qism, Toshkent-1995.