UCHBURCHAK MAVZUSIGA OID BA'ZI OLIMPIADA MASALALARNING YECHIMLARI

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ANNOTATSIYA

Mazkur maqolada ko'p uchraydigan uchburchak mavzusiga oid ba'zi olimpiada masalalarning yechimlari keltirilgan. Bular orqali o'quvchi olimpiada masalalarini bir xil usulda emas balki, boshqacha kreativ fikrlash orqali ham yechishi mumkinligini o'rganadi.

Kalit so'zlar: Uchburchak, tengsizlik, isbot, kosinuslar teoremasi, perimetri, bissektrisa, burchak.

ABSTRACT

This article provides solutions to some Olympic problems in analytic geometry (triangles). Having familiarized themselves with the solutions to these Olympic problems, students master various methods of solving, as well as develop creative thinking.

Keywords: Triangle, inequality, proof, theorem of cosines, perimeter, bisector, angle.

KIRISH

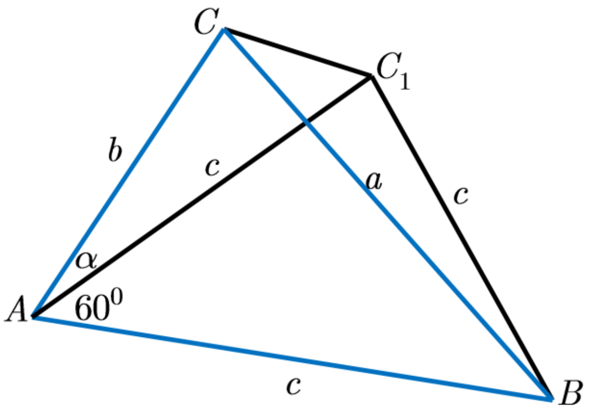
Ushbu maqola olimpiadada ishtirok etish va g'olib bo'lish istagidagi iqtidorli talabalar uchun yaratilgan. Maqola mustaqil o'rganuvchilar uchun qulay bo'lib, undagi ko'pgina masalalarning yechimlari bilan berilgan.

Maqola orqali bilimingizni boyitib, uchburchak mavzusiga oid masalalarni osongina yecha olsanggiz, olimpiadalarda g'olib bo'lsangiz, bundan xursandmiz.

ADABIYOTLAR TAHLILI VA METODOLOGIYA

1. ABC uchburchakning AB tomoniga teng tomonli ABC_1 uchburchak shunday yasalganki, uning C va C_1 uchlari AB to‘g‘ri chiziqning bir tomonida joylashgan. Ushbu $|CC_1| = \frac{1}{2}(a^2+b^2+c^2) - 2\sqrt{3}S$ tenglikni isbotlang. Bu yerda a, b, c-uchburchak tomonlari va S-uchburchak yuzasi.

ISBOT. α orqali $\angle CAC_1$ ni belgilaylik. U holda $S_{ABC} = \frac{bc\sin(60^0+\alpha)}{2}$ ekanidan va ΔACC_1 da kosinuslar teoremasiga ko‘ra:



$$|CC_1|^2 = b^2+c^2-2bc\cos\alpha = b^2+c^2-2bc\cos(60^0+\alpha-60^0) =$$

$$= b^2+c^2-2bc\left(\cos 60^0\cdot\cos(60^0+\alpha)+\sin 60^0\cdot\sin(60^0+\alpha)\right) =$$

$$= b^2+c^2-2bc\left(\frac{1}{2}\cos(60^0+\alpha)+\frac{\sqrt{3}}{2}\sin(60^0+\alpha)\right) =$$

$$= b^2+c^2-\frac{2bc\cos(60^0+\alpha)}{2}-\frac{2\sqrt{3}bc\sin(60^0+\alpha)}{2} =$$

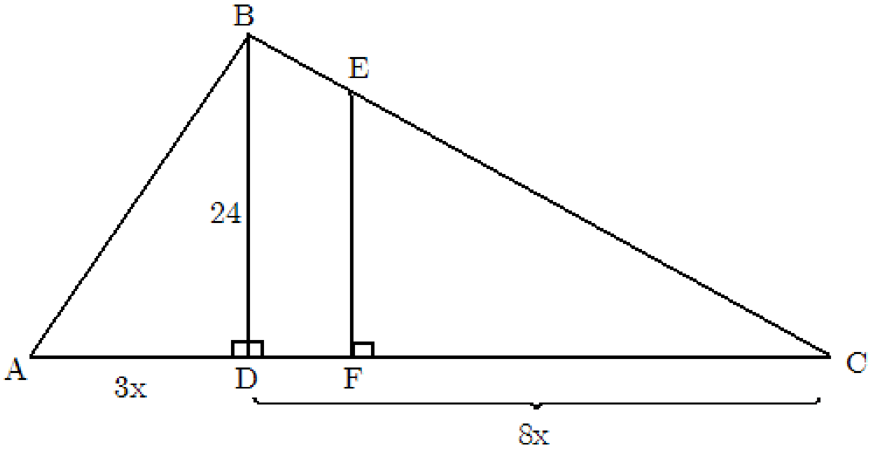
$$= \frac{2b^2+2c^2-2bc\cos(60^0+\alpha)}{2}-2\sqrt{3}S =$$

$$= \frac{b^2+c^2+b^2+c^2-2bc\cos(60^0+\alpha)}{2}-2\sqrt{3}S = \frac{a^2+b^2+c^2}{2}-2\sqrt{3}S$$

Demak, $\left|CC_1\right|^2 = \dfrac{a^2+b^2+c^2}{2} - 2\sqrt{3}S$. Isbot tugadi.

2. ABC uchburchakning BD balandligi($BD = 24$) AC tomonni A uchidan boshlab hisoblaganda 3:8 nisbatda bo‘ladi. Shu balandlikka parallel va uchburchak yuzini teng ikkiga bo‘luvchi kesma uzunligini toping.

YECHISH: Qulaylik uchun $AD = 3x$ va $DC = 8x$ deb belgilab olamiz.



U holda $S_{ABC} = \dfrac{1}{2}\cdot 11x\cdot 24 = 132x$, $S_{ABD} = \dfrac{1}{2}\cdot 3x\cdot 24 = 36x$ ekanligini topish mumkin. Shartga ko‘ra $S_{EFC} = \dfrac{S_{ABC}}{2} = \dfrac{132x}{2} = 66x$, bundan $S_{BEFD} = 66x - 36x = 30x$ va $S_{BDC} = 66x + 30x = 96x$ ekanligi kelib chiqadi. Bundan tashqari ΔBDC va ΔEFC lar o‘xshash ekanligidan: $\dfrac{S_{BDC}}{S_{EFC}} = \left(\dfrac{BD}{EF}\right)^2 \Rightarrow \dfrac{96x}{66x} = \left(\dfrac{24}{EF}\right)^2 \Rightarrow EF = 24\cdot\sqrt{\dfrac{66}{96}} = 6\sqrt{11}$ ekani kelib chiqadi.

Javob: $6\sqrt{11}$

3. Ixtiyoriy ABC uchburchak uchun $h_a \le \sqrt{p\left(p-a\right)}$ tengsizlikni isbotlang. Bu yerda h_a-uchburchakning $BC = a$ tomoniga tushirilgan balandligi, p-yarim perimetri.

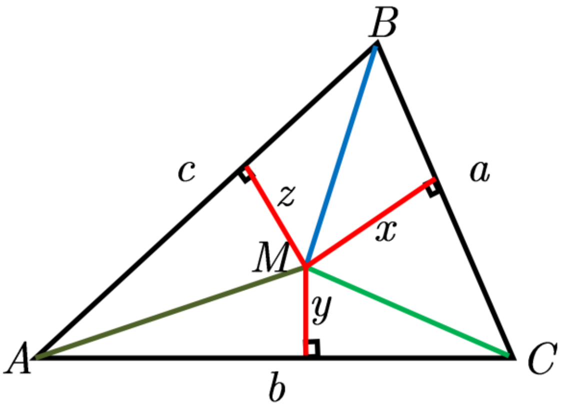
ISBOT. Nomanfiy bo‘lgan x va y sonlari uchun ushbu $(\sqrt{x}-\sqrt{y})^2 \ge 0 \Rightarrow \sqrt{xy} \le \dfrac{x+y}{2}$ tengsizlikning o‘rinli ekanidan va uchburchakning balandligini topish formulasidan foydalanamiz:

$$h_a = \frac{2}{a}\sqrt{p(p-a)(p-b)(p-c)} = \frac{2}{a}\sqrt{p(p-a)}\cdot\sqrt{(p-b)(p-c)} \le$$
$$\le \frac{2}{a}\sqrt{p(p-a)}\cdot\frac{p-b+p-c}{2} = \frac{2}{a}\sqrt{p(p-a)}\cdot\frac{2p-b-c}{2} =$$
$$= \frac{2}{a}\sqrt{p(p-a)}\cdot\frac{a+b+c-b-c}{2} = \frac{2}{a}\sqrt{p(p-a)}\cdot\frac{a}{2} = \sqrt{p(p-a)}.$$

Bundan $h_a \le \sqrt{p(p-a)}$ tengsizlikka ega bo‘lamiz. Tengsizlikda tenglik sharti $a=b=c$ bo‘lganda ya’ni, muntazam uchburchakda bajariladi. Shuni isbotlash talab qilingan edi. Isbot tugadi.

4. ABC uchburchakning ichidagi M nuqta qanday joylashganda $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\frac{a+b+c}{r}$ tenglik o‘rinli bo‘ladi? Bu yerda a,b,c-uchburchakning tomonlari, x,y,z-mos ravishda M nuqtadan BC, AC, AB tomonlargacha bo‘lgan masofalar.

YECHISH. Quyidagi chizmadan foydalanamiz:



$$S_{AMB} = \frac{1}{2}cz,\ S_{AMC} = \frac{1}{2}by,\ S_{CMB} = \frac{1}{2}ax \text{ va } S_{ABC} = \frac{1}{2}ah_a = \frac{1}{2}bh_b = \frac{1}{2}ch_c$$

ekani ma’lum. Bundan $\frac{x}{h_a}+\frac{y}{h_b}+\frac{z}{h_c}=\frac{S_{AMB}+S_{AMC}+S_{CMB}}{S_{ABC}}=1$ ekanligi kelib chiqadi.

U holda $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)\left(\frac{x}{h_a}+\frac{y}{h_b}+\frac{z}{h_c}\right)$ ko‘paytmaga Koshi-Bunyakovskiy tengsizligini qo‘llaymiz:

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\left(\frac{a}{x}+\frac{b}{y}+\frac{c}{z}\right)\left(\frac{x}{h_a}+\frac{y}{h_b}+\frac{z}{h_c}\right)\geq\left(\sqrt{\frac{a}{h_a}}+\sqrt{\frac{b}{h_b}}+\sqrt{\frac{c}{h_c}}\right)^2=$$

$$=\left(\frac{a}{\sqrt{2S}}+\frac{b}{\sqrt{2S}}+\frac{c}{\sqrt{2S}}\right)^2=\frac{(a+b+c)^2}{2S}=\frac{(a+b+c)^2}{2\cdot\frac{1}{2}(a+b+c)r}=\frac{a+b+c}{r}$$

Koshi-Bunyakovskiy tengsizligida tenglik sharti $\frac{\frac{a}{x}}{\frac{x}{h_a}}=\frac{\frac{b}{y}}{\frac{y}{h_b}}=\frac{\frac{c}{z}}{\frac{z}{h_c}} \Rightarrow \frac{ah_a}{x^2}=\frac{bh_b}{y^2}=\frac{ch_c}{z^2} \Rightarrow x=y=z=r$ bo‘lganda bajariladi. U holda M nuqta uchburchakning bissektrisalari kesishgan nuqtasi ekanini topamiz.

Javob: Uchburchakning bissektrisalari kesishgan nuqtada joylashganda.

Mustaqil ishlash uchun misol va masalalar

5. Agar uchburchakda $36S^2=(a^2+b^2+c^2)(h_a^2+h_b^2+h_c^2)$ tenglik o‘rinli bo‘lsa, uning burchaklarini toping. Bu yerda a,b,c-uchburchakning tomonlari, h_a,h_b,h_c-mos ravishda shu tomonlarga tushirilgan balandliklar, S-uchburchakning yuzi.

6. Teng yonli uchburchakning yon tomoniga o‘tkazilgan bissektrisa yon tomonni uchidan boshlab AK va KC kesmalarga ajratadi. Agar $AK+BK=BC$ tenglik o‘rinli bo‘lsa, uchburchakning burchaklarini toping.

7. O‘tkir burchakli ABC uchburchakka ichki va tashqi chizilgan aylana radiuslari mos r va R ga teng bo‘lib, $\angle BAC=\alpha$ bo‘lsa, uchburchakning yuzini toping.

8. Agar ABC uchburchakka tashqi chizilgan aylana markazi O va bu uchburchak medianalari kesishgan nuqtasi M bo‘lsa, ushbu $\overrightarrow{OM}=\frac{1}{3}(\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC})$ tenglikni isbotlang.

9. ABC uchburchakning ichida ixtiyoriy M nuqta olingan va bu nuqtadan AM, BM, CM to‘g‘ri chiziqlar o‘tkazilgan. Bu to‘g‘ri chiziqlar uchburchak tomonlarini, mos ravishda A_1,B_1,C_1 nuqtalarda kesib o‘tadi. Quyidagi tengsizlikni isbotlang:

$$\frac{AM}{A_1M}+\frac{BM}{B_1M}+\frac{CM}{C_1M}\geq 6$$

10. ABC uchburchakda $AB=c, BC=a, AC=b$ va $\angle A=30^0, \angle B=50^0$ bo‘lsa, a ni b va c orqali ifodalang.

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