

## HAJMLARNI VA YUZALARNI HISOBLASH UCHUN BITTA UNIVERSAL FORMULA HAQIDA

**Temur Rustabekovich Madraximov**

Urganch davlat universiteti Fizika-matematika fakulteti “Matematik tahlil” kafedrası  
o’qituvchisi

**Islombek Farhodovich To'raxonov**

Urganch davlat universiteti Fizika-matematika fakulteti “Matematik tahlil” kafedrası  
o’qituvchisi

**Shahzod Shuxrat o'g'li Fayzullayev**

Urganch davlat universiteti Fizika-matematika fakulteti “Matematik tahlil” kafedrası  
o’qituvchisi

### ANNOTATSIYA

Ushbu maqolada fazoviy jismlar hajmini va tekislikda ba’zi ko’pburchaklarni yuzini hisoblashda Simpson formulasining tatbiqlari ochib berilgan.

**Kalit so’zlar:** Simpson formulasi, fazoviy jism, prizma, silindr, konus, piramida, kesik konus, kesik piramida, shar, parallelogram, uchburchak, trapetsiya.

### ABSTRACT

This paper describes the application of the Simpson formula in calculating the volume of spatial objects and the faces some polygons in a plane

**Keywords:** Simpson’s formula, spatial body, prism, cylinder, cone, pyramid, truncated cone, truncated pyramid, sphere, parallelogram, triangle, trapezoid.

### KIRISH

Geometriyani o’qitishdan maqsad-tekislikdagi va fazodagi shakllarning xossalari sistemali ravishda o’rgatish va bu xossalarni hisoblash yo’li bilan yechiladigan hamda konstruktiv xarakterdagi masalalarni yechishda qo’llanish yo’li bilan o’quvchining fazoviy tasavvurlarini, mantiqiy tafakkurlarini rivojlantirish, hosil qilingan bilimlarni yer ustidagi o’lchashda, har xil qurilmalarni sirtlarini va hajmlarini aniqlashda va shuning kabi amaliy ishlarni bajarishda foydalanishni o’rgatishdir.

## ADABIYOTLAR TAHLILI VA METODOLOGIYA

Sistemali ravishda masalalar yechib borish nazariyani ongli va puxta o'zlashtirishga yordam beradi, uning amaliy qiymatini ko'rsatadi, shu bilan birga masala yechish o'quvchini mantiqiy tafakkurini, ijodiy tashabbuskorliklarini, faxm-farosatlarini, tarbiyalaydi hamda ularga bir qancha zarur amaliy mahorat va malakalar beradi.

Biz geometriyadan masalalar yechish jarayonida ko'plab fazoviy jismlarning hajmini topish muammolariga duch kelamiz. Bulardan turli prizmalar, silindr, konus, piramidalar, kesik konus, kesik piramida, shar kabi jismlar ko'p uchraydi. Ma'lumki prizma hajmi  $V=SH$ , piramida hajmi  $V = \frac{1}{3}SH$ , konus hajmi ham  $V = \frac{1}{3}SH$ , kesik piramidani  $V = \frac{1}{3}H(S_1 + \sqrt{S_1S_2} + S_2)$ , kesik konus hajmi  $V = \frac{1}{3}\pi H(R^2 + R \cdot r + r^2)$ , shar hajmi esa  $V = \frac{4}{3}\pi R^3$  formulalar orqali hisoblanadi. O'z ozidan savol tug'iladi yuqoridagi fazoviy jismlarning barchasi uchun umumiy bo'lgan formula bormi?

Bu ajoyib formula Simpson formulasi deb nomlanadi va u quyidagicha:

$$V = \frac{H}{6}(S_{quyi} + 4S_{o'rta} + S_{yuqori}),$$

bu yerda:

$S_{quyi}$ -pastki asosi yuzi,

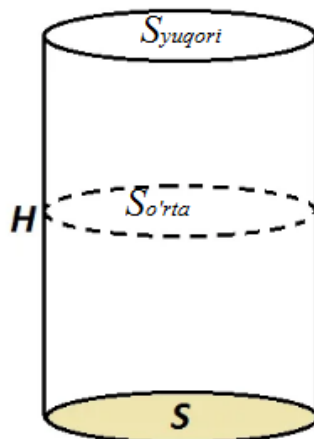
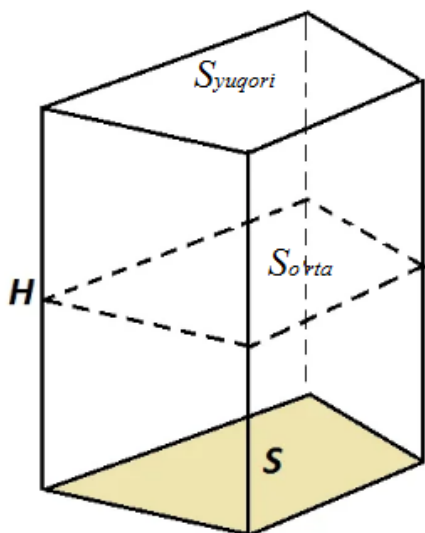
$S_{o'rta}$ - o'rta kesim yuzi,

$S_{yuqori}$ - yuqori asosi yuzi,

H-balandligi (ba'zi hollarda H ni o'rniga  $H_{max}-H_{min}$  ayirma olinadi).

Endi yuqoridagi jismlar hajmlarini Simpson formulasi orqali topamiz:

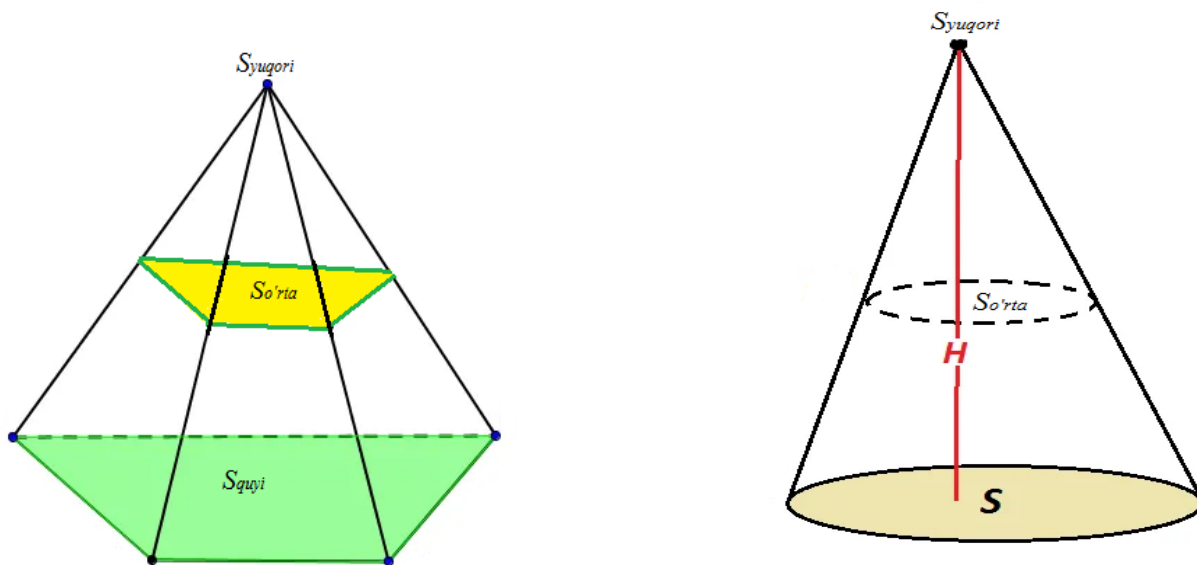
1) Prizma va silindr holida



$$S_{quyi} = S_{o'rta} = S_{yuqori} = S_{asos}$$

Demak  $V = \frac{H}{6} (S_{asos} + 4S_{asos} + S_{asos}) = S_{asos}H$

2) Piramida va konus uchun:  $S_{o'rta} = \frac{S_{asos}}{4}$ ,  $S_{yuqori} = 0$

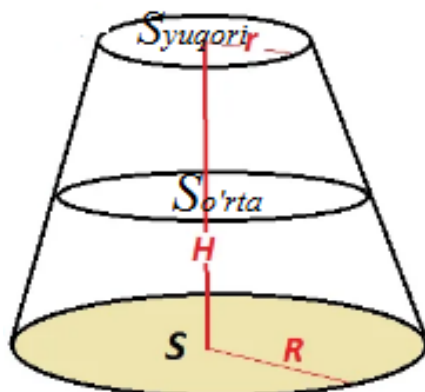


Demak  $V = \frac{H}{6} (S_{asos} + 4 \cdot \frac{S_{asos}}{4} + 0) = \frac{S_{asos} \cdot H}{3}$

3) Kesik konus holi uchun

$$S_{quyi} = \pi R^2, S_{yuqori} = \pi r^2, S_{o'rta} = \pi \left(\frac{R+r}{2}\right)^2$$

Demak,

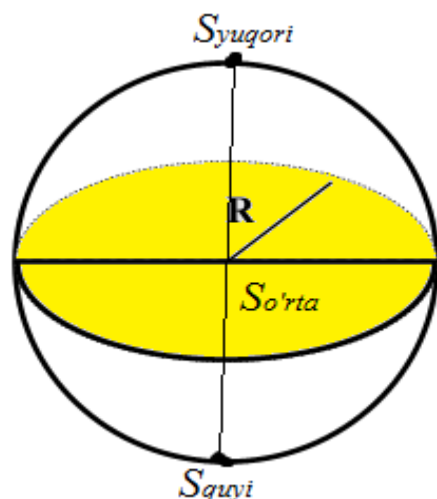


$$V = \frac{H}{6} (\pi R^2 + 4 \cdot \pi \left(\frac{R+r}{2}\right)^2 + \pi r^2) = \frac{H}{6} (\pi R^2 + \pi(R^2 + 2Rr + r^2) + \pi r^2)$$

$$= \frac{H}{6} (2\pi R^2 + 2Rr + 2\pi r^2) = \frac{1}{3} \pi H (R^2 + R \cdot r + r^2)$$

4) Endi fazoviy jism shar bolganda Simpson formulasi yordamida hajmini hisoblaymiz.

Sharning radiusi R bo'lsin, u holda:

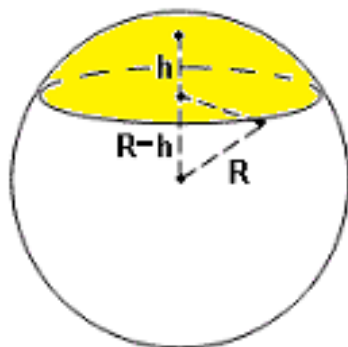


$$S_{quyi} = 0, S_{o'rtta} = \pi R^2, S_{yuqori} = 0, H = 2R$$

$$V = \frac{H}{6} (S_{quyi} + 4S_{o'rtta} + S_{yuqori}) = \frac{2R}{6} (0 + 4 \cdot \pi R^2 + 0) = \frac{8\pi R^3}{6} = \frac{4\pi R^3}{3}. \quad \text{Demak}$$

$$\text{sharning hajmi: } V = \frac{4}{3} \pi R^3$$

5) Shar segmenti hajmini topamiz:



$$S_{yuqori} = 0,$$

$$S_{quyi} = \pi(R^2 - (R-h)^2) = \pi(2Rh - h^2),$$

$$S_{o'rtta} = \pi \left( R^2 - \left( R - \frac{h}{2} \right)^2 \right) = \pi \left( Rh - \frac{h^2}{4} \right),$$

$$\text{Demak, } V = \frac{h}{6} (2\pi Rh - \pi h^2 + 4\pi Rh - \pi h^2) = \pi R h^2 - \frac{1}{3} \pi h^3$$

Endi bu formulani tekislikdagi ba'zi ko'pburchaklar uchun tatbig'ini ko'rib chiqamiz.

Simpson formulasi faqatgina fazoviy jismlar hajmi uchungina emas, bu formulaning yana bir ajoyib xususiyati



shundan iboratki undan uchburchak, parallelogramm, trapetsiyalar yuzlarini topishda foydalansa bo'ladi.

Simpson formulasi  $V = \frac{H}{6} (S_{quyi} + 4S_{o'rta} + S_{yuqori})$  dan tekislikdagi yassi ko'pburchaklar uchun yuza formulalarini keltirib chiqarishda  $V$  hajmi  $S$  yuzaga almashtiramiz,  $S_{quyi}$  ni  $a_1$  orqali belgilab quyi asosini hosil qilgan kesma uzunligi,  $S_{o'rta}$  ni  $a_2$  orqali belgilab o'rta kesma uzunligi,  $S_{yuqori}$  ni  $a_3$  orqali belgilab yuqori kesma uzunligi deb olamiz,  $h$  –figura balandligi. U holda Simpson formulasi yuzalar uchun quyidagi ko'rinishni oladi

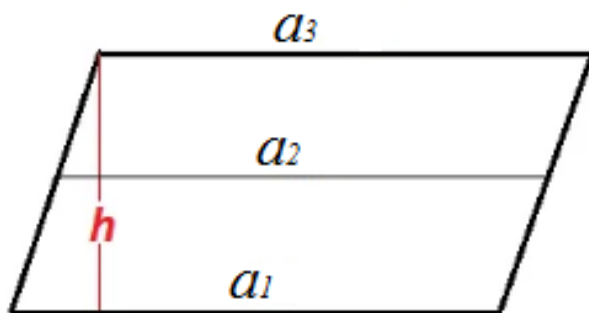
$$S = \frac{h}{6} (a_1 + 4a_2 + a_3)$$

bu yerda  $a_1, a_2, a_3$  kattalaiklar kesma yoki nuqtadan iborat bo'ladi.

Dastlab figuramiz parallelogramm (kvadrat, to'g'ri to'rtburchak) bo'lganda  $a_1 = a_2 = a_3$  bo'ladi va parallelogramm yuzini Simpson formulasi orqali hisoblasak quyidagiga teng bo'ladi

$$S = \frac{h}{6} (a_1 + 4a_2 + a_3) = \frac{h}{6} (a_1 + 4a_1 + a_1) = a_1 h$$

Demak, parallelogramm yuzi:  $S = a_1 h$



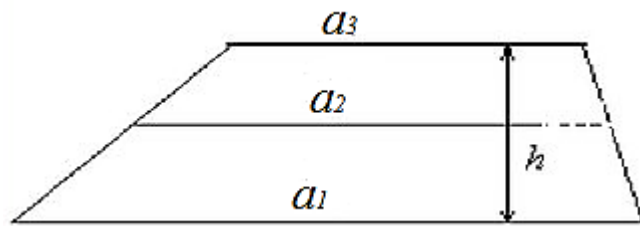
Endi Simpson formulasini uchburchak yuzini topish uchun tatbiq qilamiz. Ma'lumki uchburchak uchun  $a_2 = \frac{a_1}{2}$  (o'rta chiziq),  $a_3 = 0$  bo'ladi (chunki nuqtada iborat).

U holda

$$S = \frac{h}{6} (a_1 + 4a_2 + a_3) = \frac{h}{6} \left( a_1 + 4 \frac{a_1}{2} + 0 \right) = \frac{a_1 h}{2}$$

Demak uchburchak yuzi:  $S = \frac{a_1 h}{2}$

Endi trapetsiya yuzini Simpson formulasi yordamida topamiz.



Trapetsiya uchun  $a_2$  o'rta chiziq bo'ladi, demak  $a_2 = \frac{a_1 + a_3}{2}$  (o'rta chiziq) bo'lib

$$S = \frac{h}{6}(a_1 + 4a_2 + a_3) = \frac{h}{6}\left(a_1 + 4 \cdot \frac{a_1 + a_3}{2} + a_3\right) = \frac{a_1 + a_3}{2} \cdot h$$

Demak trapetsiya yuzi  $S = \frac{a_1 + a_3}{2} \cdot h$  yani trapetsiyaning yuzi asoslari uzunliklari yig'indisining yarmi bilan balandligining ko'paytmasiga teng.

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