KOMPLEKS ANALIZDA BA'ZI SOHALARNING YUQORI YARIM TEKISLIKKA KONFORM AKSLANTIRISHGA DOIR MISOLLAR

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ANNOTATSIYA

Mazkur maqolada kompleks analizning muhim va ko'p uchraydigan masalalaridan ba'zi sohalarning yuqori yarim tekislikka konform akslantirishga doir misollar yechimlari bilan keltirilgan.

Kalit so'zlar: konform akslantirish, tizimli tahlil, element, yuqori yarim tekislik.

ABSTRACT

This article presents some important and frequently encountered problems of complex analysis with examples of solving the conformal mapping of some domains onto the upper half-plane.

Keywords: conformal reflection, structural analysis, element, upper half-plane.

Konform akslantirish (lot. conformis — o‘xshash) — bir sohani (sirtning bir bo‘lagini yoki fazoning bir qismini) ikkinchi sohaga shunday o‘zaro bir qiymatli akslantirishki, bunda birinchi sohaning soha chegarasida yotmaydigan uchi biror a alfa burchak ostida kesishuvi ixtiyoriy (har qanday) ikkita chizig‘i ikkinchi sohaning (obrazning) shu a burchak ostida kesishuvchi ikkita chizig‘iga o‘tadi. Bunda cheksiz kichik figuralar o‘zlariga o‘xshash cheksiz kichik figuralarga almashinadi. Konform akslantirishdan xaritagrafiyada shar sirti bir qismini tekislikka tasvirlashda qadimdan foydalanib kelinadi. Konform akslantirish borasidagi ta’limot (kompleks o‘zgaruvchili funksiyalar nazariyasining bo‘limlaridan biri) gidrodinamika va aeromexanika, elastiklik nazariyasi va boshqa sohalarda keng qo‘llaniladi.

Quyida ba`zi murakkab komform akslantirishlarga doir misollarni ko’rib chiqamiz:

1-Masala. Ushbu $D = \{z \in \mathbb{C} : |z| < 1, |z-i| > 1\}$ sohani yuqori yarim tekislikka konform akslantiruvchi $w(z)$ funksiyani toping.

Yechish: Ushbu $E_1 = \{z \in \mathbb{C}: |z| < 1\}$, $E_2 = \{z \in \mathbb{C}: |z-i| > 1\}$, $D = E_1 \cap E_2$ belgilashlarni olaylik.

Ikkita $\partial E_1 = \{z \in \mathbb{C} : |z| = 1\}$, $\partial E_2 = \{z \in \mathbb{C} : |z-i| = 1\}$ aylanalarni kesishish nuqtalarini topamiz:

$$\begin{cases} |z| = 1 \\ |z-i| = 1 \end{cases} \Rightarrow \begin{cases} x^2 + y^2 = 1 \\ x^2 + (y-1)^2 = 1 \end{cases} \Rightarrow \begin{matrix} x^2 + y^2 = x^2 + (y-1)^2 \\ 2y = 1 \end{matrix} \quad y = \frac{1}{2} \quad \begin{matrix} x^2 + \left(\frac{1}{2}\right)^2 = 1 \\ x = \pm\frac{\sqrt{3}}{2} \end{matrix}$$

$$\begin{cases} z_1 = -\frac{\sqrt{3}}{2} + \frac{i}{2} \\ z_2 = \frac{\sqrt{3}}{2} + \frac{i}{2} \end{cases}$$

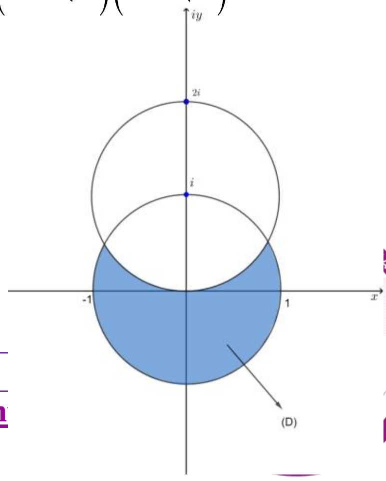
w_1 akslantirishni shunday olaylikki z_1 nuqta 0 ga, z_2 esa cheksiz uzoqlashgan nuqtaga akslansin.

$$w_1 = \frac{z - z_1}{z - z_2} = \frac{z - \left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right)}{z - \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)}$$

$z_1 = -i \in E_1$, $z_2 = 0 \in E_2$ nuqtalarini olaylik. Bu nuqatalar uchun ushbu

$$w_1(z_1) = w_1(-i) = \frac{-i - \left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right)}{-i - \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)} = \frac{\frac{\sqrt{3}}{2} + \frac{3i}{2}}{-\frac{\sqrt{3}}{2} + \frac{3i}{2}} = \frac{3i - \sqrt{3}}{3i + \sqrt{3}} = \frac{\left(3i - \sqrt{3}\right)^2}{\left(3i + \sqrt{3}\right)\left(3i - \sqrt{3}\right)} = \frac{-9\left(-\frac{\sqrt{3}}{2} + \frac{3i}{2}\right)}{-9-3} =$$

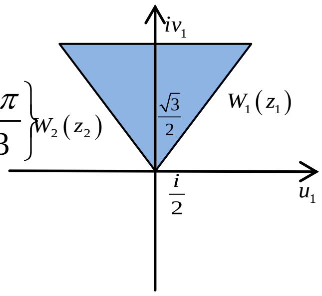
$$= \frac{i - \sqrt{3}}{-12} = \frac{1}{2} + \frac{\sqrt{3}}{2}i$$



$$w_1(z_2) = w_1(0) = \frac{0 - \left(-\frac{\sqrt{3}}{2} + \frac{i}{2}\right)}{0 - \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)} = \frac{i - \sqrt{3}}{i + \sqrt{3}} = -\frac{1}{2} + \frac{\sqrt{3}}{2} i$$

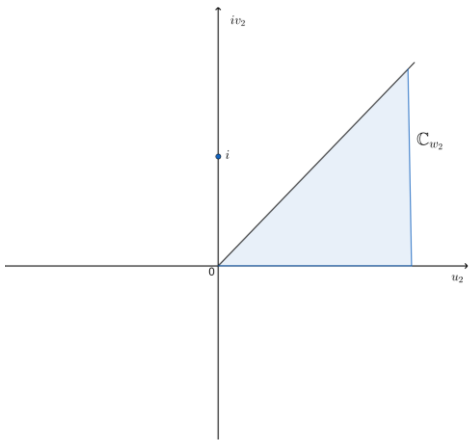
munosabatlar va ikkita aylana uchun ham $z_1 \in (E_1 \cap E_2) \to 0$ yani $w_1(z_1) = 0$ tengliklar o'rinli bo'ladi.

Bundan ko'rinadiki $D = \{z \in \mathbb{C} : |z| < 1, |z - i| > 1\}$ soha w_1 akslantirish yordamida quyidagi $D_1 = \left\{\frac{\pi}{3} < \arg W_1 < \frac{2\pi}{3}\right\}$ sohaga akslanadi.

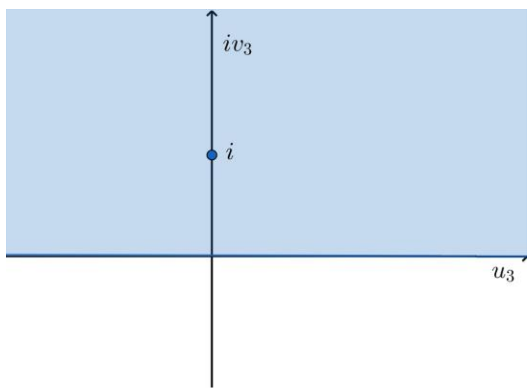


$w_2 = {}^{-\frac{\pi}{3}\bullet} w_1$ akslantirish yordamida D_1 sohani

$D_2 = \left\{W_2 \in \mathbb{C} : 0 < \arg w_2 < \frac{\pi}{3}\right\}$ sohaga akslantiramiz



$w = (w_2)^2$ darajali funksiya esa D_2 sohani $D = \{W \in \mathbb{C} : \operatorname{Im} W > 0\}$ sohaga akslantiradi



Demak $w = (w_2)^3 = \left(e^{-\frac{\pi}{3}} w_1\right)^3 = e^{-\pi i} w_1^3 = e^{-i\pi}\left(\frac{2z + \sqrt{3} - i}{2z - \sqrt{3} - i}\right)^3$ ya'ni $w = e^{\pi}\left(\frac{2z + \sqrt{3} - i}{2z - \sqrt{3} - i}\right)^3$ akslantirish $D = E_1 \cap E_2 = \{z \in \mathbb{C} : |z| < 1\}$ sohani $D^* = \{w \in \mathbb{C} : \operatorname{Im} W > 0\}$ yuqori yarim tekslikga akslantiruvchi konform akslantirishdir.

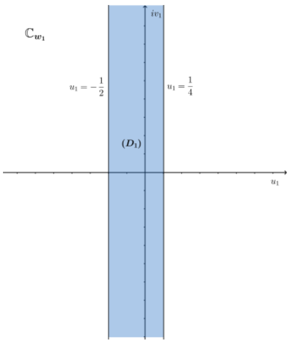
2-Masala. Ushbu $D = \ z \in \mathbb{C} : \left|z+1\right| > 1, \left|z-2\right| > 2$ sohani yuqori yarim tekislikka konform akslantiruvchi $w(z)$ funksiyani toping.

Yechish: Ushbu

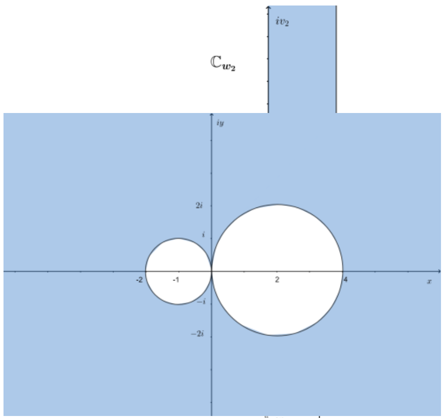
$E_1 = \{z \in \mathbb{C} : |z+1| > 1\}$, $E_2 = \{z \in \mathbb{C} : |z-2| > 2\}$, $D = E_1 \cap E_2$ belgilashlarni olaylik.

Ikkita $\partial E_1 = \{z \in \mathbb{C} : (z+1) = 1\}$ $\partial E_2 = \{z \in \mathbb{C} : |z-2| = 2\}$ aylanalarni qaraylik. Bu aylanalar $z_1 = 0$ kesishadi.

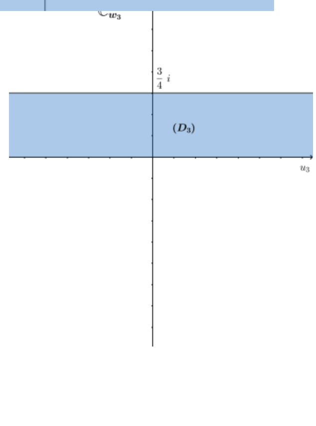
$w_1 = \dfrac{1}{z - z_1}$ akslantirishlarni qaraymiz.



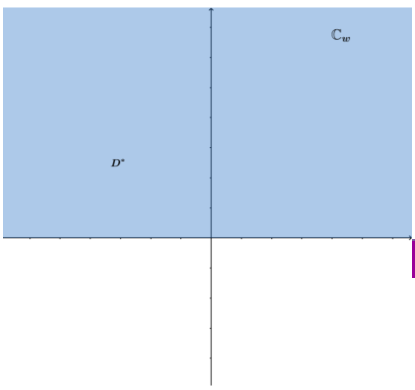
Bu akslantirish ∂E_1 aylananini $u_1 = -\dfrac{1}{2}$ to'gri chiziqqa ∂E_2 aylanani $u = \dfrac{1}{4}$ to'g'ri chiziqqa akslantiradi.



$w_2 = w_1 + \dfrac{1}{2}$ akslantirish $u_1 = -\dfrac{1}{2}$ to'g'ri chiziqni $u_2 = 0$ to'g'ri chiziqqa, $u_1 = \dfrac{1}{4}$ to'g'ri chiziqni $u_2 = \dfrac{3}{4}$ to'gri chiziqqa akslantiradi.



$w_3 = e^{i\frac{\pi}{2}} w_1$ akslantirish $u_2 = 0$ to'g'ri chiziqni $v_3 = 0$ to'g'ri chiziqqa, $u_2 = \dfrac{3}{4}$ to'g'ri chiziqni $v_3 = \dfrac{3}{4}$ to'g'ri chiziqqa akslantiradi. Ya'ni $w_3 = e^{i\frac{\pi}{2}} w_1$ akslantirish D_2 to'plamni $D_3 = \left\{z \in \mathbb{C} : 0 < \operatorname{Im} w_3 < \dfrac{3}{4}\right\}$ to'plamga akslantiradi.



$w = e^{\frac{4\pi}{3} w_3}$ akslantirishni qaraylik. Bu akslantrish D_3 to'plamni $D^* = \{z \in \mathbb{C} : \operatorname{Im} w > 0\}$ ya'ni yuqori yarim tekislikka akslantiradi.

Demak $w = e^{\frac{4\pi}{3} w_3} = e^{\frac{4\pi i}{3} w_2} = e^{\frac{4\pi i}{3}}\left(\frac{1}{z}+\frac{1}{2}\right)$ ya’ni $w = e^{\frac{4\pi i}{3}}\left(\frac{1}{z}+\frac{1}{2}\right)$ akslantirish $D = \ z \in \mathbb{C} : \left|z+1\right| > 1, \left|z-2\right| > 2$ sohani $D^* = \{z \in \mathbb{C} : \operatorname{Im} w > 0\}$ yuqori yarim tekislikka konform akslantiradi.

Mustaqil ishlash uchun misollar.

1. $|z|>1,\ |z-i|<1$ sohani yuqori yarim tekislikka konform akslantiruvchi $w(z)$ funksiyani toping.

2. $|z|>2,\ \left|z-\sqrt{2}\right|<\sqrt{2}$ sohani yuqori yarim tekislikka konform akslantiruvchi $w(z)$ funksiyani toping.

3. $D=\left\{z\in\mathbb{C}:\ |z-1|>1,\ \text{Re}z{>}0\right\}$ sohani yuqori yarim tekislikka konform akslantiruvchi $w(z)$ funksiyani toping.

4. $D=\left\{z\in\mathbb{C}:\ |z-i|>1,\ |z-2i|<2\right\}$ sohani yuqori yarim tekislikka konform akslantiruvchi $w(z)$ funksiyani toping.

5. $y=x$ va $y=x+b$ to’g’ri chiziqlari orasidagi yo’lakni yuqori yarim tekislikka konform akslantiring.

REFERENCES

1. A.Sa’dullayev, G.Xudoyberganov, H.Mansurov, A.Vorisov, T.To’ychiyev “Matematik analiz kursidan misol va masalalar to’plami” III qism. Toshkent, “O’zbekiston” 2000.

2. G.Xudoyberganov, A.Vorisov, H.Mansurov “Kompleks analiz”. Toshkent, “UNIVERSITET” 1998.

3. Б.В.Шабат “Введение в комплексный анализ”. Москва, “НАУКА” 1985.

4. T.T.To‘ychiyev, J.K.Tishabayev, D.X.Djumabayev, A.M.Kitmanov “Kompleks o‘zgaruvchili funksiyalar nazariyasi fanidan mustaqil ishlar”. Toshkent, “MUMTOZ SO‘Z” 2018.

5. Л.И.Волковыский, Г.Л.Лунц, И.Г.Араманович “Сборник задач по теории функций комплексного переменного” Учебник пособие.
ФИЗМАТЛИТ, 2002. 312 с