NUMERICAL METHODS FOR FINDING FIXED AND PERIODIC POINTS OF SOME TWO DIMENSIONAL DYNAMICAL SYSTEMS

Shavkat Jumabaevich Seytov
PhD, senior teacher, Tashkent State University of Economics
seytovsh@tsue.uz

Samad Nishonovich Nishonov
Senior teacher, Termez State University
samad.nishonov@bk.ru

ABSTRACT

In this paper, numerical methods are proposed for finding fixed and periodic points for some two-dimensional discrete dynamical systems. It made it possible to make computer calculations using numerical methods and to publish the results through the program.

Keywords. Fixed point, periodic point, numerical methods.

INTRODUCTION

We consider numerical methods of finding the fixed points of mapping given in this paper. Therefore, in this section, we give methods of approximate solution of polynomial equations. Consider an algebraic equation of $n-$ th degree $(n \geq 1)$

$$P(x) \equiv a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-1} x + a_n = 0, \qquad (1)$$

where coefficients $a_0, a_1, a_2, \ldots, a_{n-1}, a_n$ $-$are real numbers and $a_0 \neq 0.$

In the general case, the variable x will be considered in as a complex number.

Theorem (The fundamental theorem of algebra.) *An algebraic equation $n-th$ degree has exactly n real or complex roots, provided that each root is counted as many times as its multiplicit.*

It is said that the root ξ of equation (1) has multiplicity m if

$$\begin{gathered} P(\xi) = P'(\xi) = P''(\xi) = \ldots = P^{(m-1)}(\xi) = 0 \\ P^{(m)}(\xi) \neq 0. \end{gathered} \qquad (2)$$

Consequence: *An algebraic equation of odd degree with real coefficients has at least one real root.*

Theorem. *Let* $A = \max\{|a_1|,|a_2|,...,|a_n|\}$, *where* a_k – *are the coefficients of equation (1). Then absolute values of all roots of the equation (1) satisfy fallowing inequality*

$$|x_k| < 1 + \frac{A}{|a_0|},$$

those the roots of this equation on the complex plane are inside the circle.

$$|x_k| < 1 + \frac{A}{|a_0|} = R.$$

Consequence. *Let* $B = \max\{|a_1|,|a_2|,...,|a_{n-1}|\}$, *where* a_k – *are the coefficients of equation (1). Then the roots of the equation (1) satisfy fallowing inequality*

$$|x_k| > \frac{1}{1 + \frac{B}{|a_0|}} = r,$$

those the roots of this equation on the complex plane are inside the ring.

Lemma. *Let the given function* f *be continuous in the closed interval* $[a,b]$. *If* $f(a) > 0$ *and* $f(b) < 0$ *then there is a point* c *in the interval* $[a,b]$ *where* $f(c) = 0$.

Main part

To find the points where the period of following mapping is equal to four, it is necessary to solve the following equations

$$\begin{cases} \left(\left(\left(x^2+c_2\right)^2+c_1\right)^2+c_2\right)^2+c_1-x=0, \\ \left(\left(\left(y^2+c_1\right)^2+c_2\right)^2+c_1\right)^2+c_2-y=0. \end{cases}$$

Among the solutions of this system of equations are also points whose periods are equal to two. To separate them and leave only the equation of four points of period, we must divide the equations in the system of equations into the following two equations accordingly

$$x^4 + 2c_2x^2 - x + c_2^2 + c_1 \text{ ва } y^4 + 2c_1y^2 - y + c_1^2 + c_2.$$

In this case, the following system of equations is formed

$$\begin{cases} x^{12}+6c_2x^{10}+x^9+(15c_2^2+3c_1)x^8+4c_2x^7+(20c_2^3+12c_1c_2+1)x^6+ \\ +(2c_1+6c_2^2)x^5+(3c_1^2+4c_2+18c_1c_2^2+15c_2^4)x^4+(1+4c_1c_2+4c_2^3)x^3+ \\ +(c_1+6c_1^2c_2+5c_2^2+12c_1c_2^3+6c_2^5)x^2+(c_1^2+2c_2+2c_1c_2^2+c_2^4)x+ \\ +c_1^6+3c_1c_2^4+2c_2^3+3c_1^2c_2^2+2c_1c_2+c_1^3+1=0, \\ \\ y^{12}+6c_1y^{10}+y^9+(15c_1^2+3c_2)y^8+4c_1y^7+(20c_1^3+12c_1c_2+1)y^6+ \\ +(2c_2+6c_1^2)y^5+(3c_2^2+4c_1+18c_2c_1^2+15c_1^4)y^4+(1+4c_1c_2+4c_1^3)y^3+ \\ +(c_2+6c_2^2c_1+5c_1^2+12c_2c_1^3+6c_1^5)y^2+(c_2^2+2c_1+2c_2c_1^2+c_1^4)y+ \\ +c_2^6+3c_2c_1^4+2c_1^3+3c_1^2c_2^2+2c_1c_2+c_2^3+1=0. \end{cases}$$

According to Abel's theorem, these equations cannot be solved analytically in the general case. Therefore, we solve it using approximate solution methods for certain values of the parameters.

For example $c_1=-0.98$ and $c_2=-0.02$ let's solve approximately..

$$\begin{cases} 0.099144+0.919616\,x-1.09315\,x^2+1.07837\,x^3+2.79415\,x^4-1.9576\,x^5+ \\ +1.23504\,x^6-0.08\,x^7-2.934\,x^8+x^9-0.12\,x^{10}+x^{12}=0, \\ \\ -0.0115392-1.07565\,y-0.417991\,y^2-2.68637\,y^3+9.57098\,y^4+5.7224\,y^5- \\ -17.5886\,y^6-3.92\,y^7+14.346\,y^8+y^9-5.88\,y^{10}+y^{12}=0. \end{cases}$$

The equations in this system of equations are not related to each other so we solve them separately.

a. First

$$0.099144+0.919616\,x-1.09315\,x^2+1.07837\,x^3+2.79415\,x^4-1.9576\,x^5+$$
$$+1.23504\,x^6-0.08\,x^7-2.934\,x^8+x^9-0.12\,x^{10}+x^{12}=0$$

We solve numerical solutions of the equation using approximate methods. To do this, we find the gap where all the solutions are located.

$$A=\max\{|a_1|,|a_2|,\ldots,|a_n|\}=2.934,\quad R=1+\frac{A}{|a_0|}=1+\frac{2.934}{1}=3.934.$$

This means that all solutions are in the interval $(-3.934, 3.934)$.

$$f = 0.099144+0.919616\,x - 1.09315\,x^2 + 1.07837\,x^3 + 2.79415\,x^4 - 1.9576\,x^5 +$$
$$+1.23504\,x^6 - 0.08\,x^7 - 2.934\,x^8 + x^9 - 0.12\,x^{10} + x^{12},$$
$$f_1 = 0.919616-2.18631x + 3.2351\,x^2 + 11.1766\,x^3 - 9.788\,x^4 + 7.41024\,x^5 -$$
$$-0.56\,x^6 - 23.472\,x^7 + 9\,x^8 - 1.2\,x^9 + 12\,x^{11},$$
$$f_2 = -0.099144 - 0.842981\,x + 0.910962\,x^2 - 0.808776\,x^3 - 1.86276\,x^4 + 1.14193\,x^5 -$$
$$-0.61752\,x^6 + 0.0333333\,x^7 + 0.978\,x^8 - 0.25\,x^9 + 0.02\,x^{10},$$
$$f_3 = -744.5 - 6379.66\,x + 6323.19\,x^2 - 5530.42\,x^3 - 14446.2\,x^4 + 7439.43\,x^5 -$$
$$-3945.68\,x^6 - 97.04\,x^7 + 7346\,x^8 - 1287x^9,$$
$$f_4 = 0.020562+0.181177\,x - 0.144408\,x^2 + 0.126777\,x^3 + 0.423907\,x^4 - 0.132206\,x^5 +$$
$$+0.085444\,x^6 + 0.01774\,x^7 - 0.201121x^8,$$
$$f_5 = 5.0731-4.04273\,x + 29.2359\,x^2 + 47.3173\,x^3 + 13.3759\,x^4 + 27.4597\,x^5 + 27.031\,x^6 + 5.8618\,x^7,$$
$$f_6 = 0.79745-1.00711\,x + 4.99726\,x^2 + 6.49981\,x^3 + 0.109407\,x^4 + 4.10101\,x^5 + 3.33102\,x^6,$$
$$f_7 = -0.329551 - 0.544621\,x - 1.2825\,x^2 + 0.140093\,x^3 - 1.28697\,x^4 - 2.87271\,x^5,$$
$$f_8 = -0.498182 + 1.88381\,x - 3.20112\,x^2 - 5.13992\,x^3 + 0.896849\,x^4,$$
$$f_9 = 10.1898-35.1446\,x + 58.6061\,x^2 + 111.845\,x^3,$$
$$f_{10} = -0.012912 - 0.0393259\,x - 0.0202422\,x^2,$$
$$f_{11} = -111.409 - 201.795\,x$$
$$f_{12} = -0.00262955$$

	$f_0 = f$	f_1	f_2	f_3	f_4	f_5	f_6	f_7	f_8	f_9	f_{10}	f_{11}	f_{12}	
-3.934	+	-	+	+	-	-	+	+	+	-	-	+	-	7
3.934	+	+	+	+	-	+	+	-	-	+	-	-	-	5
														2

We can see that there are two real solutions to the equation.

Algorithm:

1. We divide the interval $(-3.934, 3.934)$ into two equal parts and check which interval has a solution using the Sturm theorem for each of these intervals. If there is only one solution in each interval, we find approximate solutions using an arbitrary one of the methods of dividing the section into two equal parts, watts, and attempts to find solutions for each interval.

2. If more than one solution is in the same interval, then we apply the Sturm theorem again by dividing the interval into three equal parts. If several more solutions remain in the same interval, we will continue to use Sturm's theorem to divide the interval into four, five, six, and so on. We stop when there is only one solution or no solution in each interval.

3. Then we find the approximate solutions using the arbitrary one of the methods of dividing the section into two equal parts, watts and attempts, to separate the intervals in which there is a solution and find the solutions in each interval.

For the second equation, we use the same algorithm.

b. The second

$$-0.0115392-1.07565y-0.417991y^2-2.68637y^3+9.57098y^4+5.7224y^5-$$
$$-17.5886y^6-3.92y^7+14.346y^8+y^9-5.88y^{10}+y^{12}=0$$

We solve numerical solutions of the equation using approximate methods. To do this, we find the gap where all the solutions are located

$$A=\max\{|a_1|,|a_2|,\dots,|a_n|\}=17.5886,\quad R=1+\frac{A}{|a_0|}=1+\frac{17.5886}{1}=18.5886.$$

This means that all solutions are in the interval $(-18.5886, 18.5886)$.

$$g=-0.0115392-1.07565y-0.417991y^2-2.68637y^3+9.57098y^4+5.7224y^5-$$
$$-17.5886y^6-3.92y^7+14.346y^8+y^9-5.88y^{10}+y^{12},$$
$$g_1=-1.07565-0.835981y-8.0591y^2+38.2839y^3+28.612y^4-105.532y^5-$$
$$-27.44y^6+114.768y^7+9y^8-58.8y9+12y^{11},$$
$$g_2=0.0115392+0.986011y+0.348326y^2+2.01478y^3-6.38065y^4-3.33807y^5+$$
$$+8.79432y^6+1.63333y^7-4.782y^8-0.25y^9+0.98y^{10},$$
$$g_3=1.11169+4.05728y+21.2208y^2-27.7251y^3-23.8725y^4+16.9743y^5+$$
$$+14.0365y^6-1.98041y^7-3.93753y^8-0.536027y^9,$$
$$g_4=15.437+53.3631y+287.127y^2-426.092y^3-274.673y^4+282.865y^5+$$
$$+155.229y^6-54.8164y^7-46.3148y^8,$$
$$g_5=-0.0107472-0.0728328y-0.125708y2+0.659926y^3-0.648162y^4+0.0203181y^5+$$
$$+0.307983y^6-0.132465y^7,$$
$$g_6=-28.6209-146.467y-466.802y^2+1191.69y^3-289.711y^4-484.562y^5+229.686y^6,$$
$$g_7=0.00719286+0.0711498y+0.152208y^2-0.242717y^3-0.0750924y^4+0.0865886y^5,$$
$$g_8=4.9153-68.943y+153.899y^2+11.987y^3-106.64y^4,$$
$$g_9=-0.00418028-0.117396y-0.00190424y2+0.125103y3,$$
$$g_{10}=-5.26161+62.781y-53.9861y^2,$$
$$g_{11}=0.0181738-0.037381y$$
$$g_{12}=-12.5005$$

	$g_0 = g$	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}	g_{12}	
-18.5886	+	-	+	+	-	+	+	-	-	-	-	+	-	7
18.5886	+	+	+	-	-	-	+	+	-	+	-	-	-	5
														2

We can see that there are two real solutions to the equation.
As a result

$$x_1 = -0.9798840171550919,\ x_2 = -0.09607531847629341$$
$$y_1 = -0.01076953317967882,\ y_2 = 0.9401726870760027.$$

This means that the four periods of a given mapping have four equal points

$$(x_1, y_1) = (-0.9798840171550919, -0.01076953317967882),$$
$$(x_1, y_2) = (-0.9798840171550919, 0.9401726870760027),$$
$$(x_2, y_1) = (-0.09607531847629341, -0.01076953317967882),$$
$$(x_2, y_2) = (-0.09607531847629341, 0.9401726870760027).$$

At these points, we examine the spectra of the mapping given. That is, we find the modulus of the values of the equations in a given system of equations at points x and y, respectively. It follows that this period is attractive because the absolute values of multiplier smaller then one.

Example 1.

$$F_{c_1 c_2} : \begin{cases} x' = y^2 + c_1, \\ y' = x^2 + c_2. \end{cases} \quad (32)$$

If $c_1 = c_2 = -1$ then all point of out site the rectangle $|x| \le \frac{1}{2}(1+\sqrt{5})$, $|y| \le \frac{1}{2}(1+\sqrt{5})$ tend to infinity. Some inside points tend to fixed points $(-1,0)$ *or* $(0,-1)$. And some inside points tend to periodic points with period two $(0,0)$ *and* $(-1,-1)$.

For example $x_0 = 0.776,\ y_0 = -0.36.$

n	x_n	y_n
$n=1$	-0,8704	-0,397824
$n=2$	-0,841736065024	-0,24240384

$n=3$	-0,941240378353254	-0,291480396837912
...	...	...
$n=10$	-0,999970282135738	-3,57147346333631E-6
$n=11$	-0,999999999987245	-5,94348453725036E-5
$n=12$	0,999999996467499	-2,551092670E-11
$n=13$	-1	-7,0650016823E-9
$n=14$	-1	0

For (32) when $c_1 = c_2 = -1$ then filled Julia set Fig. 1.

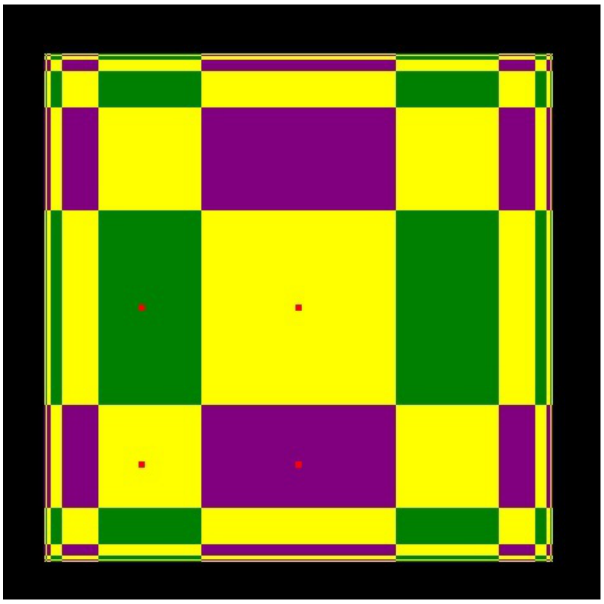

Figure 1. For $c_1 = c_2 = -1$ the classification of Julia set.

Let $c_1 = -0.98, c_2 = -0.02$ then all point in Julia set tend to the periodic points with period four

For example $x_0 = 0.06, y_0 = -0.36$.

n	x_n	y_n
$n=1$	-0,97973104	0,70318016
...	...	...
$n=16$	0,0960753164810756	-0,0107895746542244
$n=17$	-0,979883585078781	-0,0107695335630612
$n=18$	-0,979884017146834	0,940171840306845
$n=19$	-0,0960769106940412	0,940172687059817
$n=20$	-0,0960753185067235	-0,0107692272314892

Let $c_1 = -1.22, c_2 = -0.38$ then all point in Julia set tend to the periodic points with period eight but there are two cyclical points with period eight.

Classification all Cauchy problems for $c_1 = -1.22, c_2 = -0.38$ on the Figure 2.

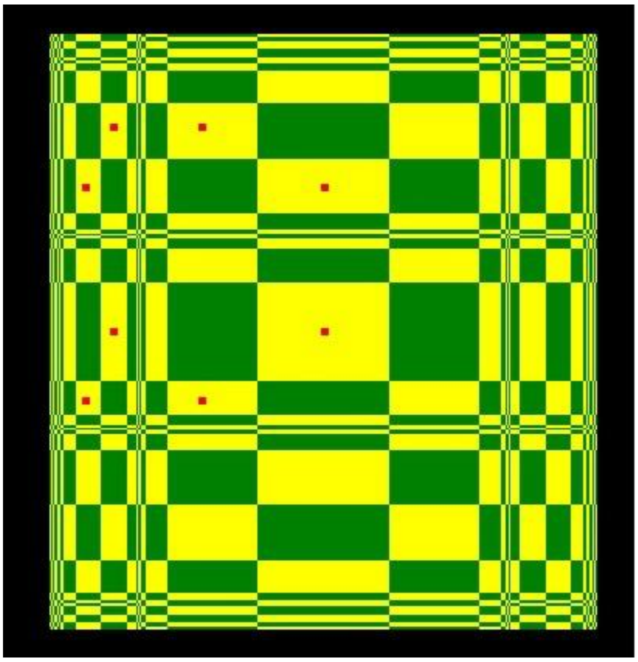

Figure 2. For $c_1 = -1.22, c_2 = -0.38$ the classification of Julia set.

Classification all Cauchy problems for $c_1 = -1.19, c_2 = -0.44.$ period 16, we get Fig. 3.

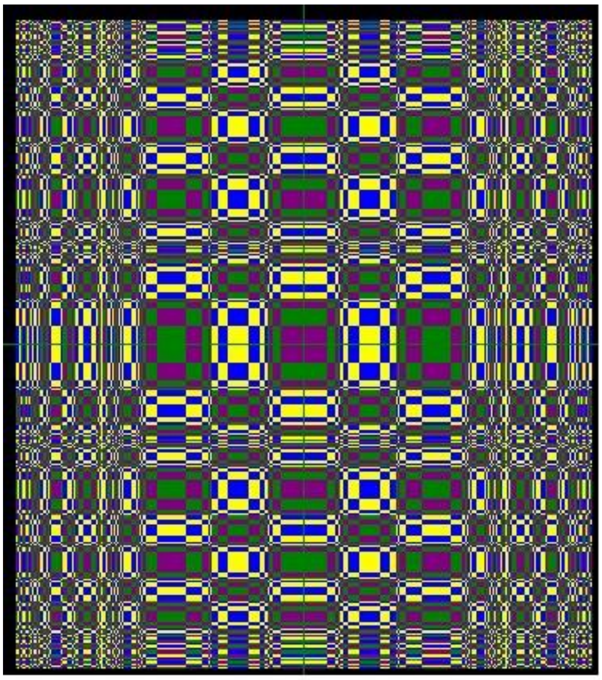

Figure 3.

Classification all Cauchy problems for $c_1 = -1.19, c_2 = -0.44$ Period 3 and 6.

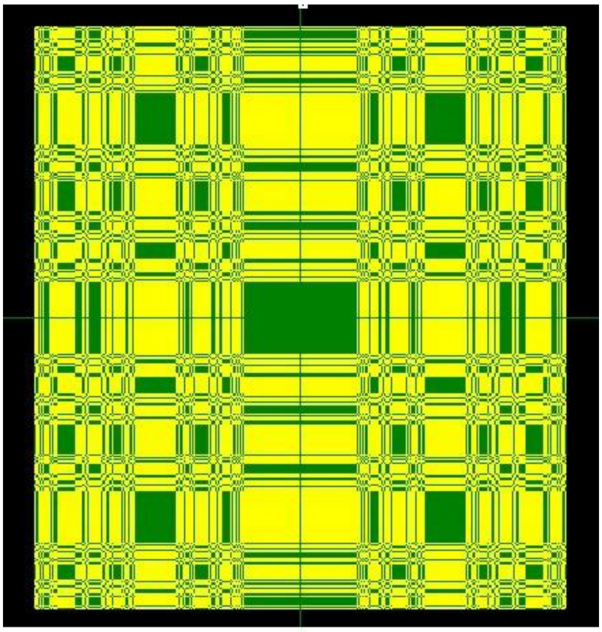

Figure 4. ***For*** $c_1 = -1.19, c_2 = -0.44$ ***the classification of Julia set.***

CONCLUSION

In this paper investigated the method of graphical analysis for some two dimensional mappings. We have classified the equilibrium, periodic and chaotic states of the model. Bifurcation curves, Julia and Mandelbrot sets for two dimensional case of the problem Von Neumann and Ulam are the main results of this section. used numerical methods for the two-dimensional case of the Von Neumann and Ulam problem. We calculated periodic points with period four by numerical methods. And the types of points were identified.

REFERENCES

1. Seytov Sh.J. *Dynamics of the populations depend on previous two steps*// Ilm sarchashmasi. 2022. Vol.1, No 1. Page. 17-22.
2. Ganikhodzhaev R.N., Seytov Sh.J. *Coexistence chaotic behavior on the evolution of populations of the biological systems modeling by three dimensional quadratic mappings* // Global and Stochastic Analysis. 2021. Vol.8, No 3. Page. 41-45. (Scopus. IF=3.8).
3. Ganikhodzhayev R.N., Seytov Sh.J. *An analytical description of Mandelbrot and Julia sets for some multi-dimensional cubic mappings* // AIP Conference Proceedings, 2021, Vol.2365, Page.050006. (Scopus. IF=0.7).
4. Ganikhodzhaev R.N., Seytov Sh.J. *Mathematical modelling of the evolutions of the populations in the connected two islands* //

Problems of computational and applied mathematics 2021. Vol.1 (31), Page.24-35.

5. Ganikhodzhayev R.N., Narziyev N.B., Seytov Sh.J. *Multi-dimensional case of the problem of Von Neumann - Ulam.* Uzbek Mathematical Journal 2015 Vol. 3, Issue 1, 11-23.

6. Seytov Sh. J., Ganikhodzhayev R. N. *The method of graphical analysis for some two dimensional dynamical systems* // Bulletin of the Institute of Mathematics, 2020, Vol.2, No 4. Page. 22-26.

7. Ganikhodzhayev R.N., Seytov Sh.J., Obidjonov I.N., Sadullayev L. *The sets of Julia and Mandelbrot for multi-dimensional case of logistic mapping* Central asian problems of modern science and education Vol. 2020, Issue 4, 81-94.