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O‘TISH MATRITSASINING XOS VEKTORLARINI ANIQLASH VA YORDAMCHI MATRITSALARNI SHAKLLANTIRISH

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ANNOTATSIYA

Fundamental matritsaning tashkil etuvchi bo‘lgan xos vektorlarning elementlarini aniqlash uchun oddiy quvish va o‘tish matritsasining algebraik to‘ldiruvchilari yordamida o‘tish matritsasining xos vektorlarining alohida elementlarini aniqlashga asoslangan yagona algoritm taklif etildi va u fundamental matritsaga teskari matritsaning elementlarini topishda ham foydalanildi.

Kalit so‘zlar: Parabolik va Giperbolik tipdagi chiziqli tenglama, Fundamental matritsa, Notriviallik sharti.

$$\begin{cases} \left(-2-\lambda_s-2\alpha_0 h\right)v_{0,s}+2v_{1,s}=0, \\ v_{p-1,s}+\left(-2-\lambda_S\right)v_{p,s}+v_{p+1,s}=0 \ \text{агар} \ \ p=1..N-1, \\ v_{N-1,s}+\left(-2-\lambda_s\right)v_{N,s}=0. \end{cases}$$ (1) dagi tenglamalar $v_{p,s}$

noma’lumlarga nisbatan bir jinsli bo‘lgani uchun, ularning qiymatlarini (1) sistemadan D'_{N+1} determinantning r satri s-chi ustunining $A_{p,s}$ algebraik to‘ldiruvchisi yordamida $v_{p,s}=c_s A_{p,s}$ o‘zgarmas ko‘paytuvchi c_s aniqligida aniqlash mumkin. Bu tasdiq chiziqli algebra [2,10,21,39] teoremalari bilan asoslanadi.

c_s ko‘paytuvchining qiymati xos vektorning

$$\sum_{p=0}^{N}\left(v_{p,s}\right)^2=1$$

normalashtirish shartidan aniqlanadi.

D'_{N+1} determinantning p-chi satri s-chi ustunini o‘chirish orqali $A_{s,p}$ algebraik to‘ldiruvchini tuzish mumkin. Lekin bu yo‘l algoritmlash uchun murakkab. Shuning uchun biz D'_{N+1} matritsaning xos vektorlarini tuzishning soddaroq variantini taklif qilamiz. $\alpha_0 l$ ning qiymatidan qat’iy nazar, $s=0$ da $A_{N,0}=\left(-1\right)^{1+N+1}\cdot 2$ va $s>0$ da esa algebraik to‘ldiruvchining

$$A_{N,s} = (-1)^{N+1+s+1}\left[(2\cos\theta_s - 2\alpha_0 h)\frac{\sin s\theta_s}{\sin\theta_s} - 2\frac{\sin(s-1)\theta_s}{\sin\theta_s}\right]$$

trigonometrik ifodasidan foydalanish mumkin.

Keyin, odatdagi quvish usuli yordamida qolgan $A_{N,s}$ algebraik to‘ldiruvchilarning qiymatlarini topamiz. Xususan, (1) sistemaning oxirgi satridan $A_{N-1,s} = (2+\lambda_s)A_{N,s}$ ga ega bo‘lamiz.

(1) ning ikkinchi satridan $p = N-2..1$ da $A_{p,s} = (2+\lambda_s)A_{p+1,s} - A_{p+2,s}$ formula kelib chiqadi. Birinchi satrdan $A_{0,s} = \dfrac{2}{2+\lambda_s + 2\alpha_0 h}A_{1,s}$ ni topamiz Biz $c_s = 1/\sqrt{\sum\limits_{k=0}^{N} A_{k,s}^2}$ ko‘paytuvchiga muvofiq o‘tish matritsasining s-chi xos vektorining elementlarini normalashtiramiz.

Shunday qilib, V matritsaning barcha komponentlari bitta algoritm bilan aniqlandi. Endi

$$V^{-1} = \begin{pmatrix} v^{-}_{0,0} & v^{-}_{0,1} & \dots & v^{-}_{0,N-1} & v^{-}_{0,N} \\ v^{-}_{1,0} & v^{-}_{1,1} & \dots & v^{-}_{1,N-1} & v^{-}_{1,N} \\ \dots & \dots & \dots & \dots & \dots \\ v^{-}_{N-1,0} & v^{-}_{N-1,1} & \dots & v^{-}_{N-1,N-1} & v^{-}_{N-1,N} \\ v^{-}_{N,0} & v^{-}_{N,1} & \dots & v^{-}_{N,N-1} & v^{-}_{N,N} \end{pmatrix}$$

matritsaning tarkibiy qismlarini aniqlashga o‘tamiz.

Ushbu matritsaning elementlarini topish uchun V matritsani teskarilash, masalan, satrning asosiy elementini tanlashga asoslangan usulidan foydalanish mumkin. Lekin biz V fundamental matritsaning elementlarini topishda yuqorida qo‘llangan protseduradan foydalanishni ma’qul ko‘rdik.

$$A = V\Lambda V^{-1} \tag{2}$$

tenglikning tomonlarini chapdan V^{-1} ga ko‘paytirish orqali

$$V^{-1}A = \Lambda V^{-1} \tag{3}$$

tenglikni hosil qilamiz.

(3) tenglik tomonlarini λ_s elementga nisbatan, ya’ni s-chi satr bo‘yicha ochib chiqamiz:

$$\left(V^{-1}A\right)_s = \Big[(-2-2\alpha_0 h)v^{-}_{s,0} + v^{-}_{s,1},\ 2v^{-}_{s,0} - 2v^{-}_{s,1} + v^{-}_{s,2}, \dots$$
$$v^{-}_{s,p-1} - 2v^{-}_{s,p} + 2v^{-}_{s,p+1}, \dots\ v^{-}_{s,N-2} - 2v^{-}_{s,N-1} + 2v^{-}_{s,N},\ v^{-}_{s,N-1} - 2v^{-}_{s,N}\Big],$$

$$\left(\Lambda V^{-1}\right)_s = \left(\lambda_s v_{s,0}^-,\ \lambda_s v_{s,1}^-,\ \ldots \lambda_s v_{s,p}^-,\ \ldots \lambda_s v_{s,N-1}^-,\ \lambda_s v_{s,N}^-\right).$$

Ushbu ikki satrning mos elementlarini taqqoslash

$$\begin{cases}\left(-2-\lambda_s-2\alpha_0 h\right)v_{s,0}^- + v_{s,1}^- = 0,\\ 2v_{s,0}^- + \left(-2-\lambda_s\right)v_{s,1}^- + v_{s,2}^- = 0,\\ v_{s,p-1}^- + \left(-2-\lambda_s\right)v_{s,p}^- + v_{s,p+1}^- = 0, \quad агар \quad p = 2..N-1,\\ v_{s,N-1}^- + \left(-2-\lambda_s\right)v_{s,N}^- = 0\end{cases} \tag{4}$$

bir jinsli tenglamalar sistemasiga olib keladi.

(4) sistemaning asosiy determinanti quyidagi ko‘rinishga ega:

$$D_{N+1}^{\prime\prime} = \begin{vmatrix} -2-\lambda_s-2\alpha_0 h & 1 & 0 & 0 & \ldots & 0 & 0 & 0 \\ 2 & -2-\lambda_s & 1 & 0 & \ldots & 0 & 0 & 0 \\ 0 & 1 & -2-\lambda_s & 1 & \ldots & 0 & 0 & 0 \\ & \ldots & & & \ldots & & \ldots & \\ 0 & 0 & 0 & 0 & \ldots & 1 & -2-\lambda_s & 1 \\ 0 & 0 & 0 & 0 & \ldots & 0 & 1 & -2-\lambda_s \end{vmatrix}_{N+1}.$$

Determinant oxirgi qatorining algebraik to‘ldiruvchilarini hisoblaymiz:

$$A'_{N,s} = (-1)^{N+1+s+1}\begin{cases} 1, \quad агар \quad s = 0,\\ \left(-2-\lambda_s-2\alpha_0 h\right)\dfrac{\sin s\theta_s}{\sin\theta_s} - 2\dfrac{\sin(s-1)\theta_s}{\sin\theta_s} \quad агар \quad s > 0.\end{cases}$$

(2.14) sistemaga ko‘ra algebraik to‘ldiruvchilarni hisoblaymiz: $A'_{N-1,s} = \left(2+\lambda_s\right)A'_{N,s}$; uchun $p = N-2..1$ $A'_{p,s} = \left(2+\lambda_s\right)A'_{p+1,s} - A'_{p+2,s}$ va $A'_{0,s} = \dfrac{A'_{1,s}}{2+\lambda_s+2\alpha_0 h}$.

Ikkinchi satr $A'_{0,s} = \dfrac{\left(2+\lambda_s\right)A'_{1,s} - A'_{2,s}}{2}$ natijani tekshirish uchun xizmat qiladi. $c'_s = 1/\sqrt{\sum\limits_{k=0}^{N}\left(A'_{k,s}\right)^2}$ ko‘paytuvchini qo‘llab, A funksiya o‘tish matritsasining s-chi xos vektori elementlarini normalashtiramiz: $v_{s,p}^- = c'_s A'_{p,s}$.

Olingan natijalarni $V^{-1}V = E$, $VV^{-1} = E$ va $V\Lambda V^{-1} = A$ tengliklarni bajarilishi orqali tekshiramiz.

Shunday qilib, o‘tish matritsasining ma’lum xos qiymatlarida biz V fundamental matritsa va unga teskari V^{-1} matritsasining elementlarini aniqlash algoritmini ishlab chiqdik. Bu bilan masalani yechishning tayyorgarlik qismi yakunlanadi va biz

$$\frac{dU}{dt}=\frac{a^2}{h^2}AU+F \tag{5}$$

tenglamaga qaytamiz.

$A=V\Lambda V^{-1}$ tenglikni hisobga olib, (5) tenglamani quyidagi ko‘rinishda yozib olamiz:

$$\frac{dU}{dt}=\frac{a^2}{h^2}V\Lambda V^{-1}U+F\,. \tag{6}$$

(2.15) ning ikkala tomonini chapdan V^{-1}ga ko‘paytiramiz. Vaqtga bog‘liq bo‘lmagan elementlar bilan matritsani differensiallash va ko‘paytirish amallarining o‘rin almashtirish xossasini hisobga olib,

$$\frac{dV^{-1}U}{dt}=\frac{a^2}{h^2}V^{-1}V\Lambda V^{-1}U+V^{-1}F$$

ga ega bo‘lamiz. Bu yerda $V^{-1}V\Lambda V^{-1}U=\left(V^{-1}V\right)\Lambda\left(V^{-1}U\right)=E\Lambda\bar{U}=\Lambda\bar{U}$.

Bundan kelib chiqadiki, agar biz

$$\bar{U}=\left(\bar{u}_0,\ \bar{u}_1,\dots,\ \bar{u}_N\right)^*=V^{-1}U=\left(\sum_{p=0}^{N}v_{\bar{0},p}u_p^{n+1},\ \sum_{p=0}^{N}v_{\bar{1},p}u_p^{n+1},\dots,\ \sum_{p=0}^{N}v_{\bar{N},p}u_p^{n+1}\right)^*, \tag{7}$$

yangi ustun-vektorni kiritsak, tenglama

$$\frac{d\bar{U}}{dt}=\frac{a^2}{h^2}\Lambda\bar{U}+\bar{F} \tag{8}$$

ko‘rinishni oladi, bu yerda

$$\bar{F}=\left(\bar{f}_0,\ \bar{f}_1,\dots,\ \bar{f}_N\right)^*=V^{-1}F=\left(\sum_{p=0}^{N}v_{\bar{0},p}F_p,\ \sum_{p=0}^{N}v_{\bar{1},p}F_p,\dots,\ \sum_{p=0}^{N}v_{\bar{N},p}F_p\right)^*,$$

F_p – F vektor-ustunning p- chi elementidir.

(8) dan quyidagi $\bar{u}_i$ ga nisbatan alohida oddiy differensial tenglamani ajratib olishimiz mumkin:

$$\frac{d\bar{u}_i}{dt}=\frac{a^2\lambda_i}{h^2}\bar{u}_i+\bar{f}_i\,. \tag{9}$$

(7) ga o‘tish, shuningdek $\bar{f}_i$ da

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} + f(x,t) \tag{10}$$

$$\text{va } u(x,0) = u^0(x) \tag{11}$$

chegaraviy shartlardan funksiyalarning ishtirok etishi bilan bog‘liq holda (9) tenglamaning ozod hadi murakkab ko‘rinishga ega bo‘lishi mumkin. Shuning uchun (9) tenglamani yechish uchun orqaga approksimatsiyalash sxemasidan foydalanish mumkin:

$$\frac{\bar{u}_i^{n+1} - \bar{u}_i^n}{\tau_n} = \frac{a^2 \lambda_i}{h^2} \bar{u}_i^{n+1} + \bar{f}_i^{n+1}, \tag{12}$$

bu yerda τ_n – t vaqt qadamining n-chi o‘zgarmas yoki o‘zgaruvchan qiymati, uning qiymati $\bar{f}_i^{n+1}$, $q_0(t)$ va $\mu_l(t)$ funksiyalar, shuningdek ularning hosilalari qiymatlarining o‘zgarish oralig‘iga ko‘ra tanlanadi.

(12) dan i-tugun uchun rekursiv munosabat hosil bo‘ladi:

$$\bar{u}_i^{n+1} = \frac{\bar{u}_i^n + \tau_n \bar{f}_i^{n+1}}{1 - a^2 \lambda_i \tau_n / h^2}. \tag{13}$$

U $n = 0, 1, \ldots$ lar uchun amalga oshiriladi. Ushbu munosabatni birinchi qo‘llash uchun kiritilgan $\bar{U}$ ustun vektoriga muvofiq hisoblangan $i = 0..N$ dagi $\bar{u}_i^0$ ning: $\bar{u}_i^0 = \sum_{p=0}^{N} \bar{v}_{i,p} u_p^0$ qiymatlari kerak bo‘ladi, bu yerda u_p^0 qiymatlar

$$u(l,t) = \mu_l(t) \tag{14}$$

boshlang‘ich shartdan olinadi.

(12) va (13) bog‘lanishlar vaqt bo‘yicha aniqlikning birinchi tartibini ta’minlaydi. Lekin (13) ifodani

$$\bar{u}_i^{n+1} = \frac{\bar{u}_i^n + \tau_n \tilde{f}_i^{n+1}}{1 - 0.5 a^2 \lambda_i \tau_n / h^2} \tag{15}$$

ko‘rinishda qabul qilish τ_n bo‘yicha aniqlikning ikkinchi tartibli yaqinlashuvini (9) ta’minlaydi. Bu yerda

$$\tilde{f}_i^{n+1} = \frac{\bar{f}_i^{n+1} + \bar{f}_i^n}{2} + \frac{a^2 \lambda_i}{h^2} \frac{\bar{u}_i^n}{2}.$$

(13) va (14) formula bo‘yicha hisob-kitoblar $\bar{u}_i^{n+1}$ga nisbatan natijaga olib keladi. u_i^{n+1} ga teskari o‘tish $u_i^{n+1} = \sum_{p=0}^{N} v_{i,p}\bar{u}_p^{n+1}$ formula bo‘yicha amalga oshiriladi.

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