MODELING CROSS-DIFFUSION PROCESSES IN MULTIDIMENSIONAL AREAS

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ABSTRACT

In this paper, we study modeling cross-diffusion processes in multidimensional areas. It is proved that for a given equation, the parameter values exist and have a numerical solution. The system of equations considered in this paper is based on most physical processes, for example, the cross-diffusion process in this system, thermal conductivity, polytrophic filtration of gas and liquid in nonlinear media is described by a source. There are a lot of partial solutions to this equation. One of the main methods of studying the problem under consideration is the construction of an integral self-similar solution. To do this, initially, when constructing a system of self-similar equations, a nonlinear subtraction method was used. The following results were obtained from this work: the front for the equation of nonlinear heat generation with doubled energy was estimated, the localization process was observed, new effects were observed, an algorithm was constructed in accordance with the obtained self-similar solution, a program code was created in the programming language, and the process modeling was visualized.

Keywords: cross-diffusion systems of equation, asymptotic solutions, new effects, self-similar and approximately self-similar solution.

KO‘P O‘LCHOVLI SOHALARDA KROSS-DIFFUZIYA JARAYONLARINI MODELLASHTIRISH

ANNOTATSIYA

Bu ishda ko'p o'lchovli sohalarda kross-diffuziya jarayonlarini modellashtirish tadqiq qilingan. Ko’rsatilgan tenglama uchun parametr qiymatlarining mavjudligi va sonli yechimga ega ekanligi isbotlangan. Ushbu ishda ko'rib chiqilgan tenglamalar sistemasi ko'pgina fizik jarayonlarga asoslangan bo’lib, masalan, bu sistemada kross-diffuziya jarayoni, issiqlik o'tkazuvchanlik, nochiziqli muhitda gaz va suyuqlikning politropik filtrlanishi manba bilan tasvirlanadi. Bu tenglamaning juda ko’plab xususiy yechimlari mavjud. Ko'rib chiqilayotgan muammoni o'rganishning asosiy usullaridan biri taqribiy avtomodel yechimini qurishdir. Buning uchun dastlab avtomodel tenglamalar sistemasini tuzishda chiziqsiz ajratish usulidan foydalanildi. Ushbu ishdan quyidagi natijalar olindi: ikki karra chiziqsiz issiqlik tarqalish tenglamasi uchun front baholandi, lokalizatsiya jarayoni kuzatildi, yangi effektlar kuzatildi, olingan avtomodel yechimga mos ravishda algoritm qurildi, dasturlash tilida dastur kodi yaratildi va jarayon vizual modellashtirildi.

Kalit so‘zlar: kross-diffuziya tenglamalar sistemasi, asimptotik yechimlar, yangi effektlar, avtomodel va taqribiy avtomodel yechimlar.

INTRODUCTION

Consider in the domain $Q=\{(t,x): t\in R_+, x\in R\}$ parabolic system of two cross-diffusion equations:

$$\begin{cases} |x|^k \dfrac{\partial u}{\partial t} = div\left(|x|^n u^{m_1-1}|\nabla u|^{p-2}\nabla u^l\right) + \gamma(t)u^{q_1}v^{r_1}|x|^k \\ |x|^k \dfrac{\partial v}{\partial t} = div\left(|x|^n v^{m_2-1}|\nabla v|^{p-2}\nabla v^l\right) + \gamma(t)u^{q_2}v^{r_2}|x|^k \end{cases} \tag{1}$$

with initial (Cauchy) condition

$$u(0,x)=u_0(x)\geq 0, \quad v(0,x)=v_0(x)\geq 0, \tag{2}$$

where $m_1>1, m_2>1, p>2, q_1,q_2,r_1,r_2\geq 1, k,l\in R$ is the parameters, $u_0(x), v_0(x)$ is the initial conditions, $|x|^n, |x|^k$ is the density of the medium, $0<\gamma(t)\in C(0,\infty)$ is the specified function.

Before constructing a self-similar solution of the system of equations (1), let us consider some cases of diffusion, for example: $m_i+p-3>0,\ i=1,2$ -the state of slow diffusion, $m_i+p-3=0, i=1,2$ -the critical state(the asymptotic is summed up depending on its solutions), $m_i+p-3<0,\ i=1,2$ is called the state of fast diffusion. An asymptotic solution is usually understood as a solution of a system of nonlinear equations that can satisfy certain conditions. Let us consider the reaction-diffusion system (1) for the case with a double nonlinear variable density and study its numerical solutions (by calculation methods). The equation (1) represents a number of physical processes [1]: the reaction diffusion process in a nonlinear environment, the heat dissipation process in a nonlinear environment, the filtration of liquid and gas in a nonlinear environment, they represent the existence of the law of polypore and other nonlinear displacements [2-3].

The Cauchy problem and boundary value problems for the equation were observed by many authors in one-dimensional and multi-dimensional cases [4-5]. The equation (1) in the processes represented by the phenomenon of finite distribution of temperature occurs [6]. In the presence of an absorption coefficient, the phenomenon of the “rear” front can occur, that is, the left front can stop after a certain time and move along the medium [7].

MAIN RESULTS AND SOLUTION METHODS

We can translate the system of equations (1) into a system of radial-symmetric equations so that we can find a solution to a self-similar or an approximately self-similar. To do this, we first introduce the notation as $r=|x|$, so that we can translate the system of equations (1) into a radial-symmetric system:

$$\begin{cases} r^k \dfrac{\partial u}{\partial t} = div\left(r^{n+N-1} u^{m_1-1} \left|\dfrac{\partial u}{\partial r}\right|^{p-2} \dfrac{\partial u}{\partial r}^{l} \right) + \gamma(t) u^{q_1} v^{r_1} r^k \\ r^k \dfrac{\partial v}{\partial t} = div\left(r^{n+N-1} v^{m_2-1} \left|\dfrac{\partial v}{\partial r}\right|^{p-2} \dfrac{\partial v}{\partial r}^{l} \right) + \gamma(t) u^{q_1} v^{r_1} r^k \end{cases} \quad (3)$$

After performing the substitution (3), to find a self-similar solution of the system of equations (1) and the solution of the approximately self-similar, we use the following method:

$$\begin{cases} u(t,r) = \bar{u}(t)\cdot \omega(\tau(t),\varphi(r)) \\ v(t,r) = \bar{v}(t)\cdot z(\tau(t),\varphi(r)) \end{cases} \quad (4)$$

Now we calculate the initial part of the system of equations (1), as required, as follows:

$$\begin{cases} \dfrac{d\bar{u}}{dt} = \gamma(t)\cdot \bar{u}^{q_1} \bar{v}^{r_1} \\ \dfrac{d\bar{v}}{dt} = \gamma(t)\cdot \bar{u}^{q_2} \bar{v}^{r_2} \end{cases} \Leftrightarrow \begin{cases} \bar{u}(t) = A_1\left[T_0 + \displaystyle\int_0^t \gamma(\eta)d\eta \right]^{\alpha_1} \\ \bar{v}(t) = A_2\left[T_0 + \displaystyle\int_0^t \gamma(\eta)d\eta \right]^{\alpha_2} \end{cases} \quad \Bigg| \quad \begin{cases} \alpha_1 = \dfrac{1-r_2+r_1}{(q_1-1)(r_2-1)-r_1 q_2} \\ \alpha_2 = \dfrac{1-q_1+q_2}{(q_1-1)(r_2-1)-r_1 q_2} \end{cases}$$

where it is equal to $A_1 = \left[\alpha_2 \alpha_1^{\frac{1-r_2}{r_1}} \right]^{-\frac{r_1}{(q_1-1)(r_2-1)-r_1 q_2}}$; $A_2 = \left[A_1^{1-q_1} \alpha_1 \right]^{\frac{1}{r_1}}$.After performing the calculations, the system of equations (3) takes the following form:

$$\begin{cases} \dfrac{\partial \omega}{\partial \tau} = \varphi^{1-s} \dfrac{\partial}{\partial \varphi}\left(\varphi^{s-1} \omega^{m_1-1} \left|\dfrac{\partial \omega}{\partial \varphi}\right|^{p-2} \dfrac{\partial \omega}{\partial \varphi}^{l} \right) + \gamma(t) \bar{u}^{q_1 (m_1+p+l-3)} \bar{v}^{r_1} \left(\omega^{q_1} z^{r_1} - \omega \right) \\ \dfrac{\partial z}{\partial \tau} = \varphi^{1-s} \dfrac{\partial}{\partial \varphi}\left(\varphi^{s-1} z^{m_1-1} \left|\dfrac{\partial z}{\partial \varphi}\right|^{p-2} \dfrac{\partial z}{\partial \varphi}^{l} \right) + \gamma(t) \bar{u}^{q_2} \bar{v}^{r_2 (m_2+p+l-3)} \left(\omega^{q_2} z^{r_2} - z \right) \end{cases} \quad (5)$$

From the system of equations (4)-(5), (1)-(2) there are important considerations on the question: if $m_i + p + l \neq 4;\ i = 1,2$

$$\tau(t) = \int_0^t [\bar{u}(\eta)]^{m_1+p+l-4} d\eta = \int_0^t [\bar{v}(\eta)]^{m_2+p+l-4} d\eta$$

or if $m_i + p + l = 4;\ i \neq 1,2$ is equal to $\tau(t) = T + t$. From these considerations, it follows that the values of arbitrary and non-zero variables:

$$\varphi(r)=\begin{cases}\dfrac{p}{p+k-n}r^{\frac{p+k-n}{p}}; n\neq p+k\\ \ln(r); n=p+k\end{cases}, s=p\frac{N+k}{p+k-n}$$

$$\begin{cases}\omega(\tau,\varphi)=f(\xi)\\ z(\tau,\varphi)=g(\xi)\end{cases} \Big| \begin{cases}\xi=\dfrac{\varphi}{\tau^{1/p}}\\ \xi_\varphi=\tau^{-1/p}\end{cases} \Leftrightarrow \begin{cases}\varphi=\xi\tau^{1/p}\\ \xi_\tau=-\dfrac{1}{p}\dfrac{\xi}{\tau}\end{cases} \tag{6}$$

After substituting (6), the system of equations (3) takes the following form

$$\begin{cases}-\dfrac{1}{p}\dfrac{\xi}{\tau}\dfrac{df}{d\xi}=\xi^{1-s}\dfrac{1}{\tau}\dfrac{d}{d\xi}\left(\xi^{s-1}f^{m_1-1}\left|\dfrac{df}{d\xi}\right|^{p-2}\dfrac{df^l}{d\xi}\right)+\gamma(t)\tau\bar{u}^{q_1-(m_1+p+l-3)}\bar{v}^{r_1}\left(f^{q_1}g^{r_1}-f\right)\\ -\dfrac{1}{p}\dfrac{dg}{d\xi}=\xi^{1-s}\dfrac{d}{d\xi}\left(\xi^{s-1}g^{m_2-1}\left|\dfrac{dg}{d\xi}\right|^{p-2}\dfrac{dg^l}{d\xi}\right)+\gamma(t)\tau\bar{u}^{q_2}\bar{v}^{r_2-(m_2+p+l-3)}\left(f^{q_2}g^{r_2}-g\right)\end{cases} \tag{7}$$

(7) by reducing (reducing) the system of equations, we form a system of equations of a new form:

$$\begin{cases}\xi^{1-s}\dfrac{d}{d\xi}\left(\xi^{s-1}f^{m_1-1}\left|\dfrac{df}{d\xi}\right|^{p-2}\dfrac{df^l}{d\xi}\right)+\dfrac{\xi}{p}\gamma(t)\tau\bar{u}^{q_1-(m_1+p+l-3)}\bar{v}^{r_1}\left(f^{q_1}g^{r_1}-f\right)=0\\ \xi^{1-s}\dfrac{d}{d\xi}\left(\xi^{s-1}g^{m_2-1}\left|\dfrac{dg}{d\xi}\right|^{p-2}\dfrac{dg^l}{d\xi}\right)+\dfrac{\xi}{p}\gamma(t)\tau\bar{u}^{q_2}\bar{v}^{r_2-(m_2+p+l-3)}\left(f^{q_2}g^{r_2}-g\right)=0\end{cases} \tag{8}$$

To find a solution to this system of equations (8), we introduce another repeating self-similar pattern:

$$\begin{cases}\bar{f}(\xi)=A\left(a_1-\xi^{\gamma_1}\right)^{\gamma_2}\\ \bar{g}(\xi)=B\left(a_2-\xi^{\gamma_3}\right)^{\gamma_4}\end{cases}$$

where $a_i\geq 0;\ i=1,2$, $\gamma_i\geq 0;\ i=1,4$ is equal.

To facilitate the calculations, we calculate the first equation of the system of equations (8):

$$\xi^{s-1}\bar{f}^{m_1-1}\left|\frac{d\bar{f}}{d\xi}\right|^{p-2}\frac{d\bar{f}^l}{d\xi}=\xi^{s-1}A^{m_1-1}\left(a_1-\xi^{\gamma_1}\right)^{\gamma_2(m_1-1)}\left|A\gamma_2\gamma_1\xi^{\gamma_1-1}\left(a_1-\xi^{\gamma_1}\right)^{\gamma_2-1}\right|^{p-2}\cdot$$
$$\cdot\left(-A^l\gamma_2 l\gamma_1\xi^{\gamma_1-1}\left(a_1-\xi^{\gamma_1}\right)^{\gamma_2(l-1)}\right)=$$
$$=-A^{m_1+p+l-3}(\gamma_2)^{l+p-1}(\gamma_1)^{l+p-1}l^l\xi^{s-1+(\gamma_1-1)(p-2)+(\gamma_1-1)}\left(a_1-\xi^{\gamma_1}\right)^{\gamma_2(m_1-1)+(\gamma_2-1)(p-2)+\gamma_2 l-1}=$$
$$=-A^{m_1+p+l-3}(\gamma_2)^{p-1}(\gamma_1)^{p-1}l\xi^{s-1+(\gamma_1-1)(p-1)}\left(a_1-\xi^{\gamma_1}\right)^{\gamma_2(l+m_1+p-3)+1-p}$$
$$\xi^{1-s}\frac{d}{d\xi}\left(\xi^{s-1}\bar{f}^{m_1-1}\left|\frac{d\bar{f}}{d\xi}\right|^{p-2}\frac{d\bar{f}^l}{d\xi}\right)+\frac{\xi}{p}\frac{d\bar{f}}{d\xi}+d_1\left(\bar{f}^{q_1}\bar{g}^{r_1}-\bar{f}\right)= \tag{9}$$
$$=-\xi^{1-s}\Big[A^{m_1+p+l-3}(\gamma_2)^{p-1}(\gamma_1)^{p-1}l(s-1(\gamma_1-1)(p-1))\xi^{s-2+(\gamma_1-1)(p-1)}\left(a_1-\xi^{\gamma_1}\right)^{\gamma_2(l+m_1+p-3)-p}-$$
$$-A^{m_1+p+l-3}(\gamma_1)^p(\gamma_2)^{p-1}l(\gamma_2(l+m_1+p-3)+1-p)\xi^{s-1+(\gamma_1-1)p}\left(a_1-\xi^{\gamma_1}\right)^{\gamma_2(l+m_1+p-3)-p}\Big]-$$
$$-\frac{1}{p}A\gamma_1\gamma_2\xi^{\gamma_1}\left(a_1-\xi^{\gamma_1}\right)^{\gamma_2}+d_1\left(A^{q_1}B^{q_1}\left(a_1-\xi^{\gamma_1}\right)^{\gamma_2 q_1}\left(a_2-\xi^{\gamma_3}\right)^{\gamma_4}-A\left(a_1-\xi^{\gamma_1}\right)^{\gamma_2}\right)=0$$

Now we will find the unknown parameters from equality (9):

$$\begin{gathered}(\gamma_1-1)p=\gamma_1 \Rightarrow \gamma_1=\frac{p}{p-1}\\ \gamma_2(l+m_1+1-3)p=\gamma_2-1 \Rightarrow \gamma_2=\frac{p-1}{l+m_1+p-4}\\ A^{m_1+p+l-3}(\gamma_1)^p(\gamma_2)^{p-1}l\frac{(1-p)}{(l+m_1+p-4)}=\frac{1}{p}A\gamma_1\gamma_2\\ A=\left[(\gamma_1)^{1-p}(\gamma_2)^{2-p}\frac{(l+m_1+p-4)}{pl(1-p)}\right]^{\frac{1}{m_1+p+l-4}}\end{gathered} \tag{10}$$

As can be seen from the calculation equations, the following parameters are found from the second equation of the system of equations (9):

$$\gamma_3=\frac{p}{p-1};\ \gamma_4=\frac{p-1}{l+m_1+p-4};B=\left[(\gamma_3)^{1-p}(\gamma_4)^{2-p}\frac{(l+m_2+p-4)}{pl(1-p)}\right]^{\frac{1}{m_2+p+l-4}}$$

Combining all the calculated equalities, we get the integral self-similar solution we are looking for:

$$\begin{cases}u_A(t,x)=\left[\alpha_2\,\alpha_1^{\frac{1-r_2}{r_1}}\right]^{-\frac{r_1}{(q_1-1)(r_2-1)-r_1q_2}}\left[T_0+\int_0^t\gamma(\eta)d\eta\right]^{\alpha_1}A\left(a_1-\xi^{\gamma_1}\right)^{\gamma_2}\\ v_A(t,x)=\left[A_1^{1-q_1}\,\alpha_1\right]^{\frac{1}{r_1}}\left[T_0+\int_0^t\gamma(\eta)d\eta\right]^{\alpha_2}B\left(a_2-\xi^{\gamma_3}\right)^{\gamma_4}\end{cases} \tag{11}$$

Since all the parameters in equality (11) for which an approximately self-similar of the solution is found, we now see an asymptotic process for some special cases. We have described the asymptotic in detail above. If, in the above equation (8), the following change is made to the process of finding a calculated self-similar solution:

$$\begin{gathered}\gamma(t)\tau\bar{u}^{q_1-(m_1+p+l-3)}\bar{v}^{r_1}\to const.,\quad t\to\infty\\ \gamma(t)\tau\bar{u}^{q_2}\bar{v}^{r_2-(m_2+p+l-3)}\to const.,\quad t\to\infty\end{gathered} \tag{12}$$

If equality (12) holds, then we can imagine how the solutions of the self-similar (11) we have found will change. The condition (12) introduced by us is now considered the asymptotic state of equality in case $\gamma(t)=const$. First, we check the system of equations (12) through a new limit condition:

$$\begin{gathered}f(0)=c_1>0,\ f(d)=0,\\ g(0)=c_2>0, g(d)=0.\end{gathered} \tag{13}$$

where is $0<d<+\infty$. (12)-(13) For the problem in $\gamma(t)=0,\ n=0,\ l=0,\ p=2$ cases, that the solutions have a trivial self-similar solution and existence properties [4], [9], [11] it is quoted in the works.

To find out in which cases fast and slow diffusion occurs in the system of equations (12), we perform a substitution in the form:

$$\begin{cases} f(\xi)=\bar{f}(\xi)y_1(\eta) \\ g(\xi)=\bar{g}(\xi)y_2(\eta) \end{cases} \Rightarrow \eta=-\ln\left(a-\xi^{\frac{p}{p-1}}\right) \tag{14}$$

were

$$\begin{cases} \bar{f}(\xi)=A\left(a_1-\xi^{\frac{p}{p-1}}\right)^{\gamma_2}=Ae^{-\gamma_2\eta} \\ \bar{g}(\xi)=B\left(a_2-\xi^{\frac{p}{p-1}}\right)^{\gamma_4}=Be^{-\gamma_4\eta} \end{cases} \tag{15}$$

or it can also be replaced with another form:

$$\begin{cases} f(\xi)=c_1\bar{f}(\xi) \\ g(\xi)=c_2\bar{g}(\xi) \end{cases} \tag{16}$$

Combining both methods, we form the following system of equations:

$$\begin{cases} \xi^{s-1}\bar{f}^{m_1-1}\left|\frac{d\bar{f}}{d\xi}\right|^{p-2}\frac{d\bar{f}^{l}}{d\xi}+\xi^{s}(\gamma\gamma_2)^{p-1}\bar{f}\in C(0,\infty) \\ \xi^{s-1}\bar{g}^{m_2-1}\left|\frac{d\bar{g}}{d\xi}\right|^{p-2}\frac{d\bar{g}^{l}}{d\xi}+\xi^{s}(\gamma\gamma_4)^{p-1}\bar{g}\in C(0,\infty) \end{cases} \tag{17}$$

or it can also be written in another view

$$\begin{cases} \xi^{1-s}\frac{d}{d\xi}\left(\xi^{s-1}\bar{f}^{m_1-1}\left|\frac{d\bar{f}}{d\xi}\right|^{p-2}\frac{d\bar{f}^{l}}{d\xi}\right)=-(\gamma\gamma_2)^{p-1}\left(s\bar{f}+\xi\frac{d\bar{f}}{d\xi}\right) \\ \xi^{1-s}\frac{d}{d\xi}\left(\xi^{s-1}\bar{g}^{m_2-1}\left|\frac{d\bar{g}}{d\xi}\right|^{p-2}\frac{d\bar{g}^{l}}{d\xi}\right)=-(\gamma\gamma_4)^{p-1}\left(s\bar{f}+\xi\frac{d\bar{g}}{d\xi}\right) \end{cases} \tag{18}$$

We will carry out this system of equations (18) separately, putting two equalities. After calculating the first equality, the following result is obtained:

$$\xi^{s-1}f^{m_1-1}\left|\frac{df}{d\xi}\right|^{p-2}\frac{df^{l}}{d\xi}=\gamma^{p-1}\xi^{s}\bar{f}L_1(y_1)$$

$$L_1(y_1)=y_1^{m_1-1}\left|\frac{dy_1}{d\eta}-\gamma_2 y_1\right|^{p-2}\left(\frac{dy_1^{l}}{d\eta}-\gamma_2 y_1^{l}\right)$$

From the second equality, the following result is obtained:

$$\xi^{s-1}g^{m_1-1}\left|\frac{dg}{d\xi}\right|^{p-2}\frac{dg^{l}}{d\xi}=\gamma^{p-1}\xi^{s}\bar{g}L_2(y_2)$$

$$L_2(y_2)=y_2^{m_2-1}\left|\frac{dy_2}{d\eta}-\gamma_4 y_2\right|^{p-2}\left(\frac{dy_2^{l}}{d\eta}-\gamma_4 y_1^{l}\right)$$

Combining the obtained results, we obtain a new system of equations:

$$\begin{cases} \xi^{1-s}\dfrac{d}{d\xi}\left(\xi^{s-1}\bar{f}^{m_1-1}\left|\dfrac{d\bar{f}}{d\xi}\right|^{p-2}\dfrac{d\bar{f}^{l}}{d\xi}\right)=(\gamma)^{p-1}\bar{f}\left[\left(s-\gamma_2\gamma_1\dfrac{\xi^{\gamma_1}}{a-\xi^{\gamma_1}}\right)L_1y_1+\gamma_1\dfrac{\xi^{\gamma_1}}{a-\xi^{\gamma_1}}\dfrac{d}{d\eta}L_1y_1\right] \\ \xi^{1-s}\dfrac{d}{d\xi}\left(\xi^{s-1}\bar{g}^{m_2-1}\left|\dfrac{d\bar{g}}{d\xi}\right|^{p-2}\dfrac{d\bar{g}^{l}}{d\xi}\right)=(\gamma)^{p-1}\bar{g}\left[\left(s-\gamma_4\gamma_1\dfrac{\xi^{\gamma_1}}{a-\xi^{\gamma_1}}\right)L_2y_2+\gamma_1\dfrac{\xi^{\gamma_1}}{a-\xi^{\gamma_1}}\dfrac{d}{d\eta}L_2y_2\right] \end{cases} \quad (19)$$

If we also consider the newly formed system of equations (19), divided into two parts, and then the following result will be obtained in the first part:

$$\frac{d}{d\eta}(L_1y_1)+\left(\frac{s}{\gamma_1}\varphi_1(\eta)-\gamma_2\right)L_1y_1+\frac{1}{p}\gamma_1^{-p}\varphi_1(\eta)\left(\frac{dy_1}{d\eta}-\gamma_2y_1\right)-\frac{r_1-r_2+1}{(q_1-1)(r_2-1)-r_1q_2}\gamma_1^{-p}\varphi_1(\eta)y_1+$$
$$+\gamma_1^{-p}\frac{e^{(-\gamma_2+r_1\gamma_4+a_1\gamma_2-1)\eta}}{a-e^{-\eta}}y_1^{q_1}y_2^{r_1}=0$$

Then the following result is formed in the second part:

$$\frac{d}{d\eta}(L_2y_2)+\left(\frac{s}{\gamma_1}\varphi_1(\eta)-\gamma_4\right)L_2y_2+\frac{1}{p}\gamma_1^{-p}\varphi_1(\eta)\left(\frac{dy_2}{d\eta}-\gamma_4y_2\right)-\frac{q_2-q_1+1}{(q_1-1)(r_2-1)-r_1q_2}\gamma_1^{-p}\varphi_1(\eta)y_2+$$
$$+\gamma_1^{-p}\frac{e^{(-\gamma_4+r_2\gamma_4+a_2\gamma_2-1)\eta}}{a-e^{-\eta}}y_1^{q_2}y_2^{r_2}=0$$

where $\varphi_1(\eta)$

$$\varphi_1(\eta)=\frac{e^{-\eta}}{a-e^{-\eta}}$$

this will be equal to. Instead of concluding, we can say that the function $\varphi_1(\eta)$ (12), (13) serves as a property of the asymptotic solution of the problem. We present two theorems that combine all the calculated results.

Theorem 1. $k_1>0$**,** $k_2>0$ let be given. Next

$$\begin{cases} f(\zeta)=h_1^0\bar{f}(\zeta)(1+o(1)) \\ g(\zeta)=h_1^0\bar{g}(\zeta)(1+o(1)) \end{cases} \quad (20)$$

the system of equations (20) will have an asymptotic at $\eta\to+\infty\left(\zeta\to a^{1-\frac{1}{p}}\right)$ point. Where it will be equal to $0<h_1^0<+\infty$ if one of the following conditions is met:

1) If condition $\delta_i>\dfrac{2-m}{p-1}$ is satisfied, then will (h_1^0,h_2^0) be the solution of the following nonlinear system of algebraic equations with roots (h_1,h_2)

$$\begin{cases} (h_2^0)^{m_1-1}(h_1^0)^{p-2}=c_1 \\ (h_1^0)^{m_2-1}(h_2^0)^{p-2}=c_2 \end{cases} \quad (21)$$

were $c_i=\dfrac{1}{p(\chi k_i)^{p-1}}, i=1,2$. By reducing the given values to the system of equations (21), we obtain the following system of equations:

$$\begin{cases} h_1^0 = \dfrac{1}{\chi k_1}\left(\dfrac{1}{\chi p k_1}\right)^{\frac{1}{p-2}}\left[\dfrac{\chi^{p-2} p k_2^{p-1}}{k_1(\chi k_1 p)^{\frac{1}{p-2}}}\right]^{\frac{m_1-2}{p}} \\ h_2^0 = \left[\dfrac{k_1(\chi k_1 p)^{\frac{1}{p-2}}}{\;}\right]^{\frac{m_2-2}{p}} \end{cases}$$

2) Given that $\delta_i = \frac{k_i - 1}{k_i}$, $i = 1,2$, (h_1^0, h_2^0) are the roots of the following nonlinear system of algebraic equations

$$\begin{cases} k_1^{p-1}(h_1^0)^{m_1+p-3} + \dfrac{a_1 (h_1^0)^{k_1-1}(h_2^0)^{k_1-1}}{a\chi^p k_1} = \dfrac{1}{p\chi^{p-1}} \\ k_2^{p-1}(h_2^0)^{m_2+p-3} + \dfrac{a_2 (h_1^0)^{k_1-1}(h_2^0)^{k_1-1}}{a\chi^p k_2} = \dfrac{1}{p\chi^{p-1}} \end{cases} \quad (22)$$

Proof. To prove Theorem 1, we use the substitution (14). When using the self-similar finding of the solution (6), the following results are obtained similarly to the substitution (14):

$$\begin{gathered} \frac{d}{d\eta} L_1(h_1,h_2) + \left(\frac{s}{\chi}\varphi(\eta) - k_1\right) L_1(h_1,h_2) + \\ + \frac{h_1' - k_1 h_1}{\chi^{p-1}}\varphi(\eta) + \frac{a_1}{\chi^p}\varphi(\eta)\left(h_1 + \varphi_1(\eta) h_1^{\delta_1}\right) = 0, \\ \frac{d}{d\eta} L_2(h_1,h_2) + \left(\frac{s}{\chi}\varphi(\eta) - k_2\right) L_2(h_1,h_2) + \\ + \frac{h_2' - k_2 h_2}{\chi^{p-1}}\varphi(\eta) + \frac{a_2}{\chi^p}\varphi(\eta)\left(h_2 + \varphi_2(\eta) h_2^{\delta_2}\right) = 0. \end{gathered} \quad (23)$$

where $\varphi(\eta) = \frac{e^{-\eta}}{a - e^{-\eta}}$, $\varphi_i(\eta) = e^{-\eta k_i(\delta_i - 1)}$, $i = 1,2.$ after performing the calculations, the system of equations (23) is compressed as follows:

$$L_1(h_1,h_2) = h_1^{m_1-1}\left(\left|\frac{dh_1}{d\eta} - k_1 h_1\right|\right)^{p-2}\left(\frac{dh_1}{d\eta} - k_1 h_1\right)$$

$$L_2(h_1,h_2) = h_2^{m_2-1}\left(\left|\frac{dh_2}{d\eta} - k_2 h_2\right|\right)^{p-2}\left(\frac{dh_2}{d\eta} - k_2 h_2\right)$$

As can be seen from the substitution (14) that we calculated, the properties of the asymptotic solution in (12) $\eta \to +\infty$ will be appropriate when there is an arbitrary $+\infty$ in the system of equations (23), since the following conditions are appropriate

$$\frac{dh_i}{d\eta} - k_i h_i \neq 0, \; h_i(\eta) > 0, \; i = 1,2.$$

First, we show that $h_1(\eta), h_2(\eta)$ are the finite roots (solutions) of the system of equations (23). We enter the markup as follows. At $\eta \to +\infty$

$$b_i(\eta)=L_i(h_1,h_2),\ i=1,2.$$

Now we can write the system of equations (23) in its new form

$$\begin{cases} b_1' = \left(\dfrac{s}{\chi}\varphi(\eta)-k_1\right)b_1+\dfrac{h_1'-k_1h_1}{\chi^{p-1}}\varphi(\eta)+\dfrac{a_1}{\chi^p}\varphi(\eta)\left(h_1+\varphi_1(\eta)h_1^{\delta_1}\right)=0 \\ b_2' = \left(\dfrac{s}{\chi}\varphi(\eta)-k_2\right)b_1'+\dfrac{h_2'-k_2h_2}{\chi^{p-1}}\varphi(\eta)+\dfrac{a_2}{\chi^p}\varphi(\eta)\left(h_2+\varphi_2(\eta)h_2^{\delta_2}\right)=0 \end{cases}$$

To make the system of equations more compact, we will make changes with new functions

$$\begin{cases} \psi_1(\upsilon_1,\eta)=-\left(\dfrac{s}{\chi}\varphi(\eta)-k_1\right)\upsilon_1-\dfrac{h_1'-k_1h_1}{\chi^{p-1}}\varphi(\eta)-\dfrac{a_1}{\chi^p}\varphi(\eta)\left(h_1+\varphi_1(\eta)h_1^{\delta_1}\right)=0 \\ \psi_2(\upsilon_2,\eta)=-\left(\dfrac{s}{\chi}\varphi(\eta)-k_2\right)\upsilon_1'-\dfrac{h_2'-k_2h_2}{\chi^{p-1}}\varphi(\eta)-\dfrac{a_2}{\chi^p}\varphi(\eta)\left(h_2+\varphi_2(\eta)h_2^{\delta_2}\right)=0 \end{cases}$$

where $\upsilon_i, i=1,2$ is a real number. The functions υ_i given in the above equation will be appropriate in any of the following conditions given for each of the $\left[\eta_{\upsilon_i},+\infty\right)\subset[\eta_0,+\infty)\left(0<\eta_0<\eta_{\upsilon_i}\right)$ intervals $\eta\in\left[\eta_{\upsilon_i},+\infty\right)$

$$\psi_i(\upsilon_i,\eta)>0,\ \psi_2(\upsilon_2,\eta)<0.$$

In the system of equations (23), the boundary of the function $b_i(\eta)$ is located on the interval $\eta\in\left[\eta_{\upsilon_i},+\infty\right)$ (taking into account the theorem), and the following data are relevant:

$$\lim_{\eta\to+\infty} b_i(\eta)<+\infty,\ \lim_{\eta\to+\infty} b_i'(\eta)=0\,.$$

From the above conclusions, we get the following result:

$$\lim_{\eta\to+\infty} h_i(\eta)=h_i^0<+\infty,\ \lim_{\eta\to+\infty} h_i'(\eta)=0\,.$$

Now we will put the obtained results in a system of substituted equations, from where we will begin the proof of the theorem:

$$\begin{cases} \lim\limits_{\eta\to+\infty} b_1'(\eta)=\lim\limits_{\eta\to+\infty}\left[-\left(\dfrac{s}{\chi}\varphi(\eta)-k_1\right)\upsilon_1-\dfrac{h_1'-k_1h_1}{\chi^{p-1}}\varphi(\eta)-\dfrac{a_1}{\chi^p}\varphi(\eta)\left(h_1+\varphi_1(\eta)h_1^{\delta_1}\right)\right]=0 \\ \lim\limits_{\eta\to+\infty} b_2'(\eta)=\lim\limits_{\eta\to+\infty}\left[-\left(\dfrac{s}{\chi}\varphi(\eta)-k_2\right)\upsilon_1'-\dfrac{h_2'-k_2h_2}{\chi^{p-1}}\varphi(\eta)-\dfrac{a_2}{\chi^p}\varphi(\eta)\left(h_2+\varphi_2(\eta)h_2^{\delta_2}\right)\right]=0 \end{cases} \quad (24)$$

Taking into account the obtained limit intervals, we pass from the system of equations (24) to the system of algebraic equations (21):

$$\begin{cases} \left(h_2^0\right)^{m_1-1}\left(h_1^0\right)^{p-2}=c_1 \\ \left(h_1^0\right)^{m_2-1}\left(h_2^0\right)^{p-2}=c_2 \end{cases}$$

$$\begin{cases} k_1^{p-1}\left(h_2^0\right)^{m_1-1}\left(h_1^0\right)^{p-2}+\dfrac{a_1\left(h_1^0\right)^{\delta_1-1}}{a\chi^p k_1}=\dfrac{1}{p\chi^{p-1}} \\ k_2^{p-1}\left(h_1^0\right)^{m_2-1}\left(h_2^0\right)^{p-2}+\dfrac{a_2\left(h_2^0\right)^{\delta_2-1}}{a\chi^p k_2}=\dfrac{1}{p\chi^{p-1}} \end{cases} \quad (25)$$

where is $\delta_i = \frac{k_i - 1}{k_i},\ i = 1,2$. It can be seen from the results obtained that we have obtained the asymptotic form (20), that is, the theorem is proved. From **Theorem 1**, we can conclude that in case $k_i < 0,\ i = 1,2$, rapid diffusion occurs. Therefore, we investigate the regular asymptotic of the fast diffusion state in the self-similar solution (20) $\zeta \to +\infty$ and determine the following limiting conditions

$$\begin{cases} f(0) = k_1 > 0,\ f(\infty) = 0 \\ g(0) = k_2 > 0,\ g(\infty) = 0 \end{cases} \tag{26}$$

In (12), we perform the following replacement

$$\begin{cases} f(\zeta) = \bar{f}(\zeta) h_1(\eta) \\ g(\zeta) = \bar{g}(\zeta) h_2(\eta) \end{cases} \tag{27}$$

all the coefficients listed here have been determined above.

Theorem 2. Solutions of the system of equations (20) under the given conditions $k_1 < 0$, $k_2 < 0$, $\delta_i > 1,\ i = 1,2$, $\eta \to +\infty (\zeta \to +\infty)$ will have the following asymptotic form

$$\begin{cases} f(\zeta) = h_1^0 \bar{f}(\zeta)(1 + o(1)) \\ g(\zeta) = h_1^0 \bar{g}(\zeta)(1 + o(1)) \end{cases} \tag{28}$$

here it will be equal to $0 < h_1^0 < +\infty$ if one of the following conditions is met:

1) If condition $(N - l)\left[(m_i - 1)(m_{3-i} - 1) - (p - 2)^2\right] - (p - n - l)(p - m_i - l) > 0$ and $a_i > \frac{p - (m_i + 1)}{(m_i - 1)(m_{3-i} - 1) - (p - 2)^2},\ i = 1,2$ are satisfied, then (h_1^0, h_2^0) will be the solution of the following system of nonlinear algebraic equations with roots (h_1, h_2)

$$\begin{cases} \left(h_2^0\right)^{m_1 - 1} \left(h_1^0\right)^{p-2} = c_1 \\ \left(h_1^0\right)^{m_2 - 1} \left(h_2^0\right)^{p-2} = c_2 \end{cases} \tag{29}$$

2) If condition $(N - l)\left[(m_i - 1)(m_{3-i} - 1) - (p - 2)^2\right] - (p - n - l)(p - m_i - l) < 0$ and $a_i < \frac{p - (m_i + 1)}{(m_i - 1)(m_{3-i} - 1) - (p - 2)^2},\ i = 1,2$ are satisfied, then (h_1^0, h_2^0) will be the solution of the following system of nonlinear algebraic equations with roots (h_1, h_2)

$$\begin{cases} (s + \chi k_1)\left(\left|k_1 h_1^0\right|\right)^{p-2} \left(h_2^0\right)^{m_1 - 1} + \frac{1}{p \chi^{p-2}} - \frac{a_1}{k_1 \chi^{p-1}} = 0 \\ (s + \chi k_2)\left(\left|k_2 h_2^0\right|\right)^{p-2} \left(h_1^0\right)^{m_2 - 1} + \frac{1}{p \chi^{p-2}} - \frac{a_1}{k_2 \chi^{p-1}} = 0 \end{cases} \tag{30}$$

Proof. We substitute (12) into the system of equations (27) initially, and we get the following form:

$$\begin{aligned} &\frac{d}{d\eta} L_i(h_1, h_2) + \left(\frac{s}{\chi}\varphi_1(\eta) - k_i\right) L_i(h_1, h_2) + \\ &+ \frac{1}{p\chi^{p-1}} \varphi_1(\eta)\left(h_1' - k_i h_i\right) + \frac{a_i}{\chi^p} \varphi_1(\eta)\left(h_1 + \varphi_{2i}(\eta) h_i^{\delta_i}\right) = 0, \end{aligned} \tag{31}$$

where is 1 $\varphi_1(\eta)=\frac{e^{-\eta}}{a-e^{-\eta}}$, $\varphi_{2i}(\eta)=e^{-\eta k_i(\delta_i-1)}\varphi_1(\eta)$, $i=1,2$, and

$$L_i(h_1,h_2)=h_{3-i}^{m_i-1}\left(\left|\frac{dh_i}{d\eta}+k_ih_i\right|\right)^{p-2}\left(\frac{dh_i}{d\eta}+k_ih_i\right).$$

As a consequence, the study of the properties of the solution of the asymptotic system of equations (14) in (31) is the same as the study of the system of equations (2) around, since the following condition is always appropriate

$$\frac{dh_i}{d\eta}+k_ih_i\neq 0,\ \ h_i(\eta)>0,\ i=1,2.$$

Thus, we pass from the system of equations (31) in $\eta\to+\infty$ to the system of algebraic equations (30) with the necessary conditions. The proof of Theorem $+\infty$ occurs in this way.

CONCLUSION

The results of computational experiments show that the iterative methods listed above would be effective in solving nonlinear problems. Nonlinear effects result if the nonlinear division method and linear self-similar solutions are used as the initial approximation of the solution, in which the standard equation is constructed in a functional way. As expected, to achieve the same accuracy, the Newton method requires less iteration compared to the Picard methods and the special method due to the successful choice of the initial approach. In each of the cases considered, the Newton method has the best approximation by choosing a good initial approximation. In some cases, the total number of iterations is almost two times less, and the maximum number of iterations is almost 4 times less than in other methods. The results of numerical calculations show the influence of the numerical speed of distortion propagation, and the localization of the resolution depends on the values of the numerical parameters. All the results of digital experiments are visualized.

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