

## UZLUKSIZ FUNKSIYANING MODULI

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## ANNOTATSIYA

Maqolada  $[a, b]$  kesmada chegaralangan  $f(x)$  funksiyaning uzluksizlik moduli va uning asosiy xossalari tahlil qilinadi.

**Kalit so'zlar:** funksiya, uzluksiz funksiya, matematik induksiya, Gyolder fazosi.

## MODULE OF CONTINUOUS FUNCTION

## ABSTRACT

The article analyzes the continuity module of the function  $f(x)$  bounded on the section  $[a, b]$  and its main properties.

**Keywords:** function, continuous function, mathematical induction, Golder space.

$[a, b]$  da chegaralangan  $f(x)$  funksiya berilgan bo'lsin. Ushbu

$$\sup_{\substack{|x_1 - x_2| < \delta \\ a \leq x_1, x_2 \leq b \\ 0 < \delta \leq b - a}} |f(x_1) - f(x_2)| \stackrel{\text{def}}{=} \omega(f, \delta)$$

funksiyaga  $f(x)$  funksiyaning modul uzluksizligi deyiladi.

Takidlaymizki,  $\lim_{\delta \rightarrow 0} \omega(f, \delta) = 0$  shart  $f(x)$  funksiyaning uzluksiz bo'lishi uchun zaruriy va yetarli shart bo'ladi.

$[a, b]$  da uzluksiz bo'lgan funksiyalar sinfini  $C_{[a, b]}$  deb belgilaymiz. Biz bundan keyin  $f(t) \in C_{[a, b]}$  deb qaraymiz.

Uzluksiz funksiyaning modul uzluksizligi ta'rifidan uning quyidagi xossalari kelib chiqadi:

1<sup>0</sup>.  $\omega(f, 0) = 0$

2<sup>0</sup>.  $\omega(f, \delta)$  funksiya  $\delta$  bo'yicha kamayuvchi.

3<sup>0</sup>.  $\omega(f, \delta)$  yarim additiv, ya'ni

$$\omega(f, \delta_1 + \delta_2) \leq \omega(f, \delta_1) + \omega(f, \delta_2)$$

4<sup>0</sup>.  $\omega(f, \delta)$ ,  $\delta$  bo'yicha  $[a, b]$  da uzluksiz funksiya bo'ladi.

**1-ta'rif.** Agar  $\omega(\delta)$  ( $0 < \delta \leq l_0 = b - a$ ) funksiya  $1^0 - 4^0$  shartlarni qanoatlantirsa, u holda uzluksiz funksiyaning modul uzluksizligi deyiladi.

**1-lemma.**  $\omega(\delta)$  ( $a, l_0]$  da  $\uparrow$ ,  $\varphi(\delta) \geq 0$ , yarim additiv bo'lsa, u holda  $\forall t_1, t_2 \in (a, l_0]$  uchun  $|\varphi(t_1) - \varphi(t_2)| \leq \varphi C|t_1 - t_2|$  o'rinli.

$\omega(\delta)$ -modul uzluksiz bo'lsin. U holda  $\omega(f, \delta) = \omega(\delta)$  bo'ladi.

Haqiqatdan ham

$$\omega(\omega, \delta) = \sup_{|\delta_1 - \delta_2| < \delta} |\omega(\delta_1) - \omega(\delta_2)| \leq \sup_{|\delta_1 - \delta_2| < \delta} \omega(|\delta_1 - \delta_2|) \leq \omega(\delta)$$

ikkinchi tomondan  $\omega(\omega, \delta) \geq \omega(\delta) - \omega(0) = \omega(\delta)$ ,

ya'ni  $\omega(\omega, \delta) = \omega(\delta)$ .

**2-lemma.** Agar  $[0, b - a]$  da kamaymovchi, uzluksiz bo'lgan  $\omega(\delta)$  funksiya:  $\omega(0) = 0$ ,  $\frac{\varphi(\delta)}{\delta}$  o'smovchi bo'lsa, u holda  $\omega(\delta)$ -modul uzluksiz bo'ldi.

**Isbot.**  $\varphi(\delta)$  funksiya uchun  $1^0, 2^0$  va  $4^0$  lar lemmaning shartiga ko'ra bajariladi.  $3^0$  munosabat quyidagi

$$\begin{aligned} \varphi(\delta_1 + \delta_2) &= \delta_1 \cdot \frac{\varphi(\delta_1 + \delta_2)}{\delta_1 + \delta_2} + \delta_2 \cdot \frac{\varphi(\delta_1 + \delta_2)}{\delta_1 + \delta_2} \leq \\ &\leq \delta_1 \cdot \frac{\varphi(\delta_1)}{\delta_1} + \delta_2 \cdot \frac{\varphi(\delta_2)}{\delta_2} = \varphi(\delta_1) + \varphi(\delta_2). \end{aligned}$$

Takidlaymizki,  $\frac{\varphi(\delta)}{\delta}$  ning o'smovchi bo'lishlik sharti modul uzluksiz uchun yetarli shart bo'lib hisoblanadi.

Modul uzluksizlikning navbatdagi xossasi:

**5<sup>0</sup>.** Agar  $\omega(\delta)$ -modul uzluksiz bo'lsa, u holda  $\forall \lambda \in \mathbb{R}$  ( $\lambda > 0$ ) uchun

$$\omega(\lambda\delta) = (\lambda + 1)\omega(\delta) \quad (1)$$

tengsizlik o'rinli bo'ladi.

**Isbot.** a)  $\forall n \in \mathbb{N}$  bo'lsin. Bu xossaning isboti  $3^0$  xossadan bevosita kelib chiqadi, ya'ni

$$\omega(\delta_1 + \delta_2) \leq \omega(\delta_1) + \omega(\delta_2) \Rightarrow \delta_1 = \delta_2 = \delta$$

deb olsak,

$$\omega(2\delta) \leq 2\omega(\delta)$$

Matematik induksiya usuli yordamida  $\omega(n\delta) \leq n\omega(\delta)$  tengsizlikning o'rinli ekanligini ko'rsatish mumkin.

b)  $\forall \lambda \in \mathbb{R}$  bo'lib, ( $\lambda > 0$ )  $n < \lambda < n + 1$  bo'lsin.

Ma'lumki,  $\omega(\delta)$ -monoton o'suvchi funksiya. U holda  $\omega(\lambda\delta) \leq \omega(n + 1)\delta \leq (\lambda + 1)\omega(\delta)$ .

**6<sup>o</sup>.**  $\delta_1 \leq \delta_2$  bo'lsin. U holda

$$\omega(\delta_2) = \omega\left(\delta_1 \cdot \frac{\delta_2}{\delta_1}\right) \leq \left(\frac{\delta_2}{\delta_1} + 1\right) \omega(\delta_1) = \frac{\delta_2}{\delta_1} \left(1 + \frac{\delta_1}{\delta_2}\right) \omega(\delta_1) \leq 2 \frac{\delta_2}{\delta_1} \omega(\delta_1).$$

Keyingi tengsizlikdan  $\forall \delta_1 \leq \delta_2$  bo'lganda

$$\frac{\omega(\delta_2)}{\delta_2} \leq 2 \frac{\omega(\delta_1)}{\delta_1} \quad (*)$$

bo'ladi. Bu xossadan  $\frac{\omega(\delta)}{\delta}$  –deyarli kamaymovchi funksiya ekanligi kelib chiqadi.

$\varphi(\delta)$   $(0, l_0]$  da aniqlangan musbat funksiya bo'lsin.

**2-ta'rif.** Agar  $\exists A > 0$  ( $A_1 > 0$ ) topilib,  $\forall 0 < \delta_1 < \delta_2$  lar uchun  $\varphi(\delta_1) \leq A\varphi(\delta_2)$  ( $\varphi(\delta_1) \geq A\varphi(\delta_2)$ ) bo'lsa, u holda  $\varphi(\delta)$  funksiya  $(0, l_0]$  da deyarli o'suvchi (deyarli kamayuvchi) deyiladi.

**3-lemma.** Agar  $\omega(\delta)$ -modul uzluksiz bo'lsa, u holda

$$\frac{1}{2} \varphi(\delta) \leq \frac{1}{8} \int_0^\delta \omega(t) dt \leq \varphi(\delta) \quad (2)$$

tengsizlik o'rinli bo'ladi.

**Isbot.**  $\omega(\delta)$ -modul uzluksiz bo'lsin.  $\Rightarrow \frac{1}{8} \int_0^\delta \omega(t) dt \leq \frac{1}{8} \int_0^\delta \omega(\delta) dt \leq \omega(\delta)$ .

$$\omega(\delta) - \frac{1}{8} \int_0^\delta \omega(t) dt \leq \frac{1}{8} \int_0^\delta (\omega(\delta) - \omega(t)) dt \leq \frac{1}{8} \int_0^\delta \omega(\tau) d\tau, \quad [\delta - t = \tau]$$

$$\frac{1}{2} \varphi(\delta) \leq \frac{1}{8} \int_0^\delta \omega(\tau) d\tau.$$

**4-lemma.** Agar  $\omega(\delta)$ -modul uzluksiz bo'lsa, u holda

$$\frac{1}{x+y} \int_0^{x+y} \omega(t) dt \leq \frac{1}{x} \int_0^x \omega(t) dt + \frac{1}{y} \int_0^y \omega(t) dt \quad (3)$$

(bunda  $0 \leq x, y, x + y \leq l_0$ ) tengsizlik o'rinli bo'ladi.

**Isbot.** (3) da  $y = \alpha x$  deb olish bilan topamiz:

$$\frac{\int_0^{(1+\alpha)x} \omega(t) dt}{1+\alpha} \leq \frac{\int_0^{\alpha x} \omega(t) dt}{\alpha} + \int_0^x \omega(t) dt \quad (4)$$

(4) ning to'g'riligi ushbu

$$\psi(x) = \int_0^x \omega(t) dt + \frac{1}{\alpha} \int_0^{\alpha x} \omega(t) dt - \frac{1}{1+\alpha} \int_0^{(1+\alpha)x} \omega(t) dt$$

funksiyaning  $x = 0$  da  $\psi(0) = 0$  bo'lishi va uning kamaymovchi ekanligidan kelib chiqadi.

**1-teorema.** Agar  $\omega(\delta)$ -modul uzluksiz bo'lsa, u holda

$$\omega^*(\delta) = \frac{1}{\delta} \int_0^\delta \omega(t) dt$$

funksiya ham modul uzluksiz bo'ladi.

**Isbot.** Agar (2) tengsizlikni e'tiborga olsak, u holda

$$\lim_{\delta \rightarrow 0} \omega^*(\delta) = 0$$

ekanligiga ishonch hosil qilish qiyin emas. 2-lemmaga asosan  $(\omega^*(\delta))' \geq 0$ , ya'ni  $\omega^*(\delta)$ -kamaymovchi funksiya. Ravshanki  $\omega^*(\delta)$  –uzluksiz.  $\omega^*(\delta)$  ning yarm additivligi 3-lemmadan kelib chiqadi. Shu bilan teorema isbot bo'ldi.

$\varphi(\delta)$  va  $\psi(\delta)$  funksiyalar  $(0, l_0]$  da aniqlangan noldan farqli, musbat uzluksiz bo'lsin.

**3-ta'rif.** Agar  $\exists A_1, A_2 > 0$  sonlar mavjud bo'lib,  $\forall \delta_1, \delta_2 \in (0, l_0]$  lar uchun

$$A_1 \psi(\delta) \leq \varphi(\delta) \leq A_2 \psi(\delta)$$

tengsizlik bajarilsa, u holda  $\varphi(\delta)$  va  $\psi(\delta)$  funksiyalar  $(0, l_0]$  da ekvivalent ( $\varphi \sim \psi$ ) deyiladi.

**4-ta'rif.** Agar  $\exists A > 0$  soni mavjud bo'lib,  $\forall 0 < \delta_1 < \delta_2 < l_0$  lar uchun

$$\varphi(\delta_1) \leq A\varphi(\delta_2)$$

tengsizlik o'rinli bo'lsa, u holda  $\varphi(\delta)$  funksiya  $(0, l_0]$  da deyarli o'suvchi deyiladi.

**5-ta'rif.** Agar  $\exists A_1 > 0$  son mavjud bo'lib,  $\forall 0 < \delta_1 < \delta_2 < l_0$  lar uchun

$$\varphi(\delta_1) \geq A_1 \varphi(\delta_2)$$

tengsizlik bajarilsa, u holda  $\varphi(\delta)$  funksiya  $(0, l_0]$  da deyarli kamayuvchi deyiladi.

Takidlaymizki, agar  $M \geq \varphi(\delta) \geq \alpha > 0$  ( $0 \leq \delta \leq 1$ ) bo'lsa, u holda u deyarli o'suvchi bo'ladi. Bu holda  $= \frac{M}{\alpha}$ .

Agar  $\varphi(\delta)$ -modul uzluksiz bo'lsa, u holda (\*) tengsizlikdan  $\frac{\varphi(\delta)}{\delta}$  ning deyarli kamayuvchi funksiya bo'lishligi kelib chiqadi.

Ravshanki, agar  $\varphi(\delta) \sim \psi(\delta)$  bo'lib,  $\varphi(\delta)$  ning deyarli o'suvchi (deyarli kamayuvchi) ligidan  $\psi(\delta)$  ning deyarli o'suvchi (deyarli kamayuvchi) ligi kelib chiqadi.

**5-lemma.**  $\varphi(\delta)$  funksiya deyarli o'suvchi (deyarli kamayuvchi) bo'lishligi uchun kamaymovchi (o'smovchi) funksiyaga ekvivalent bo'lishi zarur va yetarlidir.

**Isbot.** Yetarliligi.  $\varphi(\delta)$  funksiya biror kamaymovchi  $\psi(\delta)$  funksiyaga ekvivalent, ya'ni  $\varphi(\delta) \sim \psi(\delta)$  bo'lsin. 2-ta'rifga ko'ra  $\exists A_1, A_2 > 0$  sonlari mavjud bo'lib,  $\forall 0 < \delta \leq l_0$  lar uchun

$$A_1 \psi(\delta) \leq \varphi(\delta) \leq A_2 \psi(\delta)$$

tengsizlik bajariladi.  $\delta_2 < \delta_1$  bo'lsin.

U holda yuqoridagi tengsizlikdan

$\varphi(\delta_2) \leq A_2 \psi(\delta_2) \leq A_2 \psi(\delta_1) \leq A_2 \frac{\varphi(\delta_1)}{A_1} = \frac{A_2}{A_1} \varphi(\delta_1)$  bo'ladi. Demak,  $\varphi(\delta)$  funksiya deyarli o'suvchi.

Zarurligi.  $\varphi(\delta)$  deyarli o'suvchi funksiya bo'lsin, ya'ni  $\forall 0 < \delta_1 < \delta_2 \in (0, l_0]$  lar uchun

$$\varphi(\delta_1) \leq A\varphi(\delta_2) \quad (5)$$

ushbu

$$\psi(\delta) = \sup_{0 < \eta < \delta} \varphi(\eta) \quad (6)$$

funksiyani tuzamiz.

Ravshanki,  $\psi(\delta)$  funksiya kamaymovchi. Endi kamaymovchi  $\psi(\delta)$  funksiyaning  $\varphi(\delta)$  funksiyaga ekvivalentligini ko'rstamiz:  $\psi(\delta)$  funksiyaning tuzulishidan  $\varphi(\delta) \leq \psi(\delta)$ . (5) dan  $\forall \eta (0 < \eta < \delta)$  uchun

$\psi(\eta) \leq A\varphi(\delta) \Rightarrow \psi(\delta) = \sup_{0 < \eta < \delta} \varphi(\eta) \leq A\varphi(\delta), \psi(\delta) \leq A\varphi(\delta)$ .

(6) ni e'tiborga olib topamiz:  $\frac{1}{A}\psi(\delta) \leq \varphi(\delta) \leq \psi(\delta)$  tengsizlik bajariladi. Keyingi tengsizlikdan  $\varphi(\delta) \sim \psi(\delta)$  ekanligi kelib chiqadi.

Xuddi shunday yo'l bilan lemmaning ikkinchi qismi ham isbot qilindi, ya'ni :  $\varphi(\delta)$  funksiya deyarli kamayuvchi bo'lishligi uchun uning biror o'smovchi funksiya ekvivalent bo'lishi zarur va yetarlidir.

Ushbu maqola umumlashgan Gyolder fazosida funksiyaning uzluksizlik moduli va uning asosiy xossalari o'rganishga bag'ishlangan.

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