COLLAPSE, REVIVAL AND QUANTUM SELF-TRAPPING OF ORBITAL ANGULAR MOMENTUM

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Abstract

Rabi oscillations and two associated effects of collapse-revival (CR) and quantum self-trapping (QST) arise in the coupling of two states. We analyze Rabi oscillations of orbital angular momentum (OAM) internally coupled fields confined in an infinite circular quantum well. As such, our model system corresponds to exciton-polaritons in etched micropillar fed by pulse excitations carrying topological charges in the linear regime; the discreteness of OAM shifts the Rabi frequency and induces the CR of OAM oscillations akin to CR in quantum optics. In nonlinear regime, interactions suppress the transfer of OAM, leading the self-trapping of AOM in one component. Imparting linear momentum too, leads to OAM randomness in the output beam. Our findings open research lines for manipulating of OAM based on non-classical states in micropillars

INTRODUCTION

One Key concept quantum realm is the quantum discreteness. In principle, it comes from a kind of constrain in considering system. The simplest example is the discreetness of eigenvalues arises from the specific boundary conditions and need to be specified explicitly (e.g. zero wave function at solid wall). Solid sates provide another notable example: a periodic lattice crystal constraint the eigenvalues to from ban energies. Another discreteness appears in angular momentum (spin or orbital), where it is restricted to half-integers or integers. The reason for such quantization lies at the associated rotational operators forming a closed group (Lie group) of any discreteness we may find an outstanding application in our every day-life, ranging from lasers and atoms to semiconductor-based computers and phones. One consequence of discreteness was shown in the problem of coupling between light and the matter, where the well-known phenomena of Rabi oscillation (OR) emerged. They are periodic features of two-level quantum system, where excitations are transferred cyclically between the two levels. The theoretical analysis of ROs begins with the Jaynes -cummings model [1], describing interaction between atom and electromagnetic field[2], considering the atom initially in excited state, the quantum states of the atom + field are $\left\{ \left| e,0 \right\rangle, \left| g.1 \right\rangle \right\}$. If atom is in the vacuum cavity (simply, kind of quantum box), the state of the atom + field in later time evolves like $\cos(\Omega_0 t)\left| e,0 \right\rangle + \sin(\Omega_0 t)\left| g,1 \right\rangle$ .where Ω_0 is the Rabi frequency. If the cavity is not vacuum but rather contains n photons, one expects similar oscillations but with shifted Rabi frequency $\Omega_0\sqrt{n+1}$. The Rabi oscillations has become ubiquitous in different parts of the physics, including atoms dynamic studies[3,4] Josephson-junction qubit[5], nuclear spin transitions[6], quantum dots[7,8] and two-component Bose-Einstein condensates[9,10] among the others. Another system which provides Rabi oscillations is micro cavity excitons polaritons in condensed matter physics [11], polaritons are bosonic quasi-particles formed by the quantum super positions of semiconductor excitons and cavity photons. They are important since of controlled-room temperature operation (duo to their photonic component) and manifesting nonlinear regimes (duo to their exciton component) [11,12] , particular features of polaritons have led to observe the Rabi oscillations with different physics. Among them we can mention ROs with decay and pumping [13], with polarization [14], with phase imbalance [15], and with linear and angular momentums [16,18], in this paper we use polariton state in micropillar to highlight some peculiar quantum aspects of the fields carrying orbital angular momentum (OAM).namely, we understand the regime of collapse and revival (CR) of orbital angular momentum (OAM) in a binary system, where Rabi oscillations are at the dynamics. Also, we propose the regime of OAM self-trapping. CR is the well-known phenomenon in quantum optics, where an atom in excited state interacts with coherent field inside a cavity[19], Due to quantum discreteness of both field and atom, the Rabi oscillation is a sum of cosine (or sine) terms oscillating at $\Omega_0\sqrt{n+1}$. Since of the coherent initial state of the field, the distribution of the Fock states are poisonings and it may lead to destructive(out of phase) or constructive(in phase) oscillations of photon numbers, respectively, correspond to collapse and revival of oscillations. Similarly, the CR of relative phase between the two condensates, observed in coherent state of atomics matter field due to collision between atoms [20]. The CR phenomena Also was reported I the single photon kerr Regime [21] and provides a testbed of field quantization [22]. In linear regime of our model we develop the regime of CR of OAM, relies on the quantum discreteness (imposed by quantum well structure of micro pillar) and the superposition of different state of the quantum well (imposed by the sending a pulse). However, quantum self-trapping (QST) is a nonlinear phenomenon occurring in binary system [23]. It refers to self-locked population imbalance between the two coupled components, where they are linked to each other either externally or through a weak barrier[24] or internally through Rabi coupled fields[15], to confirm the proposed phenomena, we study the exciton-polaritons in a confined geometry of pillar micro cavity. Here we have all essential ingredients to understand CR and QST in our model, namely: (1) the discreteness of the field inside the microPillar and (2) repulsive interaction between the particles duo exciton- exciton interaction. The particles carrying orbital angular momentum are injected through pulses, and then the system is in the superposition of different eigenstates of circular quantum well. As the result, the expectation value of OAM develops a series collapses and revivals due confinement. Going to the regime of many body interactions, the QST of OAM is also observed. Also, we extend our analysis to the moving wave pockets, where the packets are forced to move via imparted linear momentum. The dynamics leads to randomness of OAM per particle in time. The paper organized as follows. In section II we develop our theoretical model. As we described, we use the physic of exciton-polariton to address our findings. We provide equations of motions of photon and exciton fields as a system of coupled equations consisting of Schrodinger and Gross pitaeski equations while the dynamics is fed by a pulse excitation in the photon field. In section III A we discuss the results: CR and QST of OAM. And, finally in the last section (IV) the conclusion remarks are given

II. THEORY

In the following lines we introduce the theoretical model of exciton-photon coupling in a micropillar. The field theory is the starting point to model the formulation of binary (two components) field, where it is described by the two complex scalar fields ψ_c and ψ_x [25]. It worth noting that the two fields are interpreted as wave functions. The dynamics can be modeled by the Hamiltonian:

$$H = \frac{\hbar^2}{2m_c}\left|\nabla\Psi_c\right|^2 + \frac{\hbar^2}{2m_x}\left|\nabla\Psi_x\right|^2 + E_C\psi_C{}^{\dagger}\psi_C \qquad (1)$$

$$+E_X\psi_X{}^{\dagger}\psi_X + g\left|\psi_x\right|^4 + \hbar\Omega\left(\psi_c^{\dagger}\psi_x + \psi_x^{\dagger}\psi_c\right),$$

Where m_c and m_x are the masses in photon (ψ_c) and exciton (ψ_x) fields, respectively. The energy landscape is engineered by $E_c \equiv E_c^{(0)} + v_c$ and $E_x \equiv E_x^{(0)} + v_x$ where $E_{c,x}^{(0)}$ characterize the minimum energy of photon and exciton fields, v_c and v_x are extremal potentials. The can be created by different method[26], for example by etching the surface[27] or by laser trapping[28]. The nonlinearity coefficient g stands for particle - particle interaction constant, and the last term indicates the Rabi coupling between the two fields, with Rabi energy given by $\hbar\Omega$. Upon introducing the Lagrangian $L = i(\psi_c^{\dagger}\partial t\psi_c + \psi_x^{\dagger}\partial t\psi_x) - H$ and varying the corresponding action with respect to $\psi_{c,x}$, one can find the main equations of motion as:

$$i\hbar\partial t\begin{pmatrix}\psi_c(x,y,z)\\ \psi_x(x,y,z)\end{pmatrix} = L\begin{pmatrix}\psi_c(x,y,z)\\ \psi_x(x,y,z)\end{pmatrix}, \qquad (2)$$

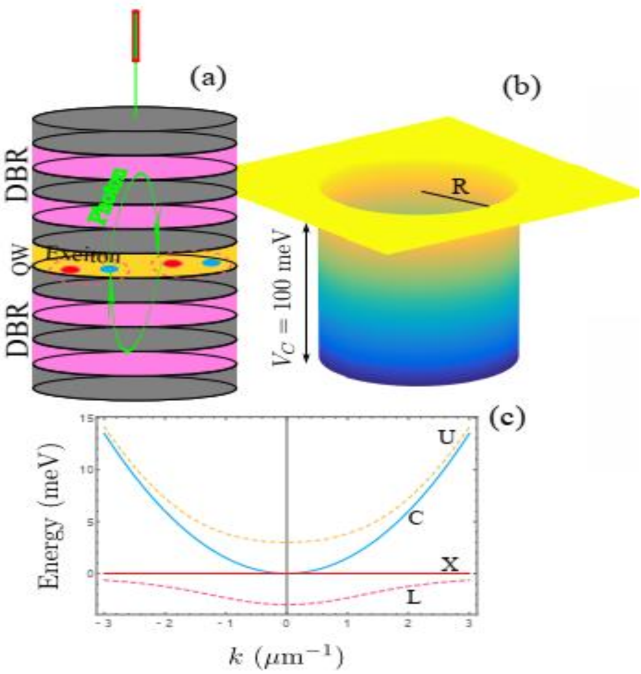

FIG.1. (a) The micrpillar structure including two sets of distributed Brag reflectors and one layer holding quantum well excitons. This layer (shown in dark yellow) has

different sickness compared to other layers, excites cavity photon. (b) An external potential (for example in etched micropillar) can confine exciton and photon states in a circular quantum well. R is the radius of quantum well. (c) The dispersion of exciton-polariton, which results in randomness of OAM. Here, C stands for photon field dispersion and has a parabolic shape. For exciton field (X), the effect mass is of several order larger than photon effective mas and its dispersion can be neglected. In strong coupling regime between photon and exciton fields, the upper (U) and lower (L) normal modes are formed. For large (k) their dispersions are being deviated from parabolic shape.

Where

$$L = \begin{pmatrix} -\frac{\hbar^2\nabla^2}{2m_c} + E_c & \hbar\Omega \\ \hbar\Omega & -\frac{\hbar^2\nabla^2}{2m_x} + E_x + g\left|\psi_x\right|^2 \end{pmatrix}. \quad (3)$$

We work in the regime of big mass imbalance $(m_x \gg m_c)$ corresponds exciton-polariton in micro cavity. The details of such structure are shown in Fig. (1). we assume the trapping potential confines photonic component ($v_x = 0$), and forms a circular infinite quantum well[26]. We solved Eq. (2) numerically and consider the dynamics for the following scheme: initially, we assume $\psi_c(x,y,0) = \psi_x(x,y,0) = 0$, and the system is pumped through pulse, with the pumping term (in polar coordinates $r = \sqrt{x^2 + y^2}$ and $\varphi = \arg(x\_iy)$):

$$p_j(x,y,t) = R_J f_j(r,\varphi) e^{-(t-t_j)^2} / 2\delta_t^2 e^{i\omega_j t}, \qquad (4)$$

added to the photon component of Eq. (2). Here,

$f_j \equiv (\frac{r}{\omega_j})^{|l_j|} e^{il_j\varphi} e^{\frac{-r^2}{2\omega_j^2}} e^{ik.r}$, l_i is the winding number of the field and R_i is the pumping amplitude. ω_j is the parameter to control the wave packet size; the pulse is being sent at time t_j with frequency ω_j and may carry linear momentum $k \equiv (k_x, k_y)$.

III. RESULTS

Our polar tonic system is fed by a pulsed pump in the photon field. The excitations then transfer between the coupled fields, lead to Rabi oscillations. In free space (no confinement) one expects the Rabi oscillations occur with no limit: the periodic trend in associated parameters (for example in amplitude and phase of the fields) with Rabi frequency. In the present section we consider how confinement and nonlinearity alter the Rabi oscillations of orbital angular momentum.

A. Collapses and revivals of OAM

In this section we work in the linear regime of the dynamics, and we set $g = 0$. Due to existence of the topological charge, here $l_1 = 1$, the fields are constrained to have zero density $\left(\left|\psi_{c,x}\right| = 0\right)$ at $(0,0)$ of 2DD space and in time evolution of the system, the core remained stationary. The limits the $\langle L_Z \rangle_{C,X} = 1$.However there are oscillations in $\langle L_Z \rangle_{C,X}$. Figure 2 shows the time evolution of $\langle L_Z \rangle$ in photon and exciton fields. Also, we define the fractional orbital angular momentum imbalance:

$$\rho_L \equiv \frac{\langle L_Z \rangle_C - \langle L_Z \rangle_X}{\langle L_Z \rangle_C + \langle L_Z \rangle_X}. \quad (5)$$

The collapse and revival of oscillations occur in the time variation $\langle L_Z \rangle$.The underlying physics is destructive or instructive interferences among different states excited inside the quantum well. One can show it by expanding $\psi_{C,X}$ based on the eigenstates of the circular infinite quantum well:

$$\psi_C = \sum_{n,l} C_{n,l} \xi_{n,l}, \quad (6a)$$

$$\psi_X = \sum_{n,l} X_{n,l} \xi_{n,l}, \quad (6b)$$

Where $\xi_{n,l} = N_{n,l} e^{il\varphi} j_l(\beta_{n,l} \frac{r}{R})$, represents the quantum states of the quantum well. Here, $N_{n,l}$ IS the normalization constant, j_l is the Bessel function of the first kind, $j_l(\beta_{n,l}) = 0$. the expectation value OAM is

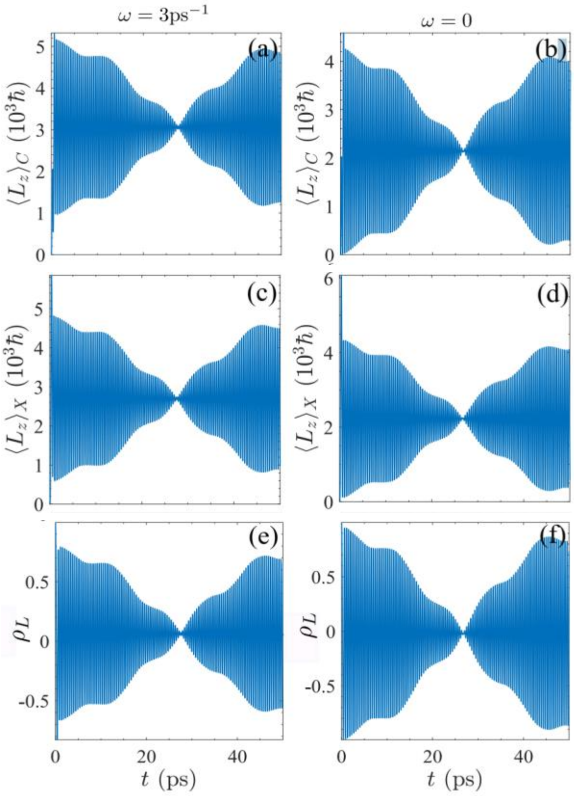

FIG.2. Collapse and revival of orbital angular momentum (OAM) in exciton-polariton confined in a circular quantum well. The time-varying OAM are shown for two energy lasers of the pump: $w = 3ps^{-1}$ (first column) and $w = 0$ (second column). The third row shows the evolution of OAM imbalance ρ_L (see the text for definition). By changing the laser energy, the average of ρ_L is

shifted from positive to negative values. We assume the size of the wave packet $w = 4\ \mu$m is less than the radius of the quantum well $R = 12\ \mu$m.
Given:

$$\langle L_Z \rangle_C = i\sum_{n,l} \delta_{l,l_1} l \left|C_{n,l}\right|^2 \frac{R^2}{2}\left[J_{l+1}(\beta_{n,l})\right]^2 \qquad (7a)$$

$$\langle L_Z \rangle_x = i\sum_{n,l} \delta_{l,l_1} l \left|x_{n,l}\right|^2 \frac{R^2}{2}\left[J_{l+1}(\beta_{n,l})\right]^2 \qquad (7b)$$

Where δ is the Kroniker delta funiction. Replacing expansions (6) in the main Eq. (2), one can find cumbersome analytical expression of $C_{n,l}$ and $X_{n,l}$ .the general form of these expressions have some common terms.

$$C_{n,l} \approx p_{n,l}^{(j)}\left[C_{n,l}^{(1)}\sin(t\Omega_{n,l}) + C_{n,l}^{(2)}\cos(t\Omega_{n,l})\right] \quad (8a)$$

$$x_{n,l} \approx p_{n,l}^{(j)}\left[x_{n,l}^{(1)}\sin(t\Omega_{n,l}) + x_{n,l}^{(2)}\cos(t\Omega_{n,l})\right] \quad (8b)$$

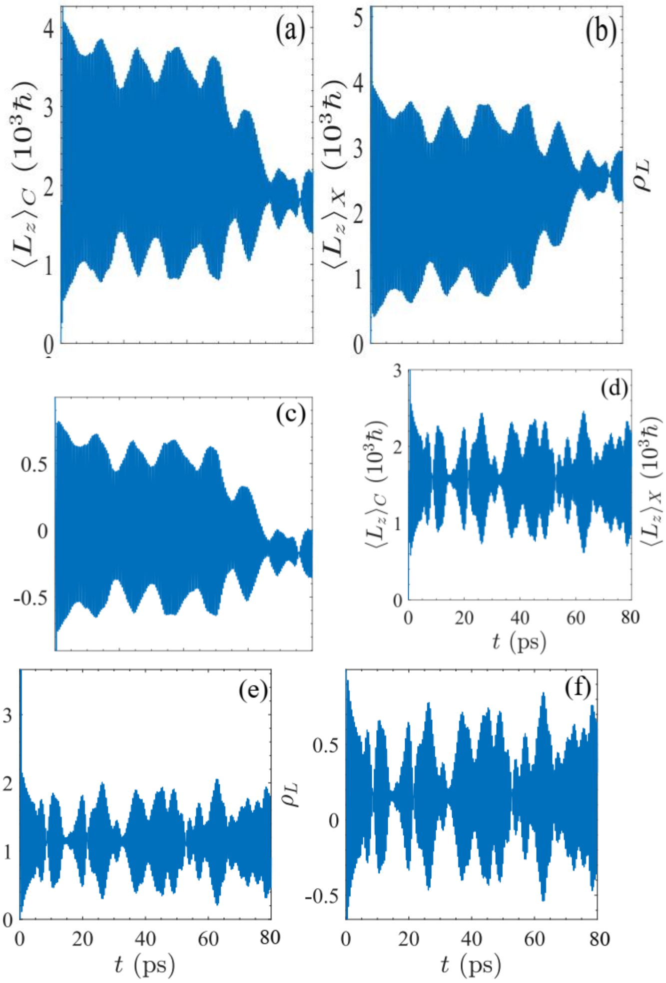

FIG. 3. (a-c) the regime of quantum self-trapping of orbital angular momentum. Here we assume $g = 20meV\,\mu m^2$ Panels (d-f) show the effect imparted momentum, where the photon field is forced to move along the x axis $(k_x = 1.2,$ $k_y = 0)$. Where $\Omega_{n,l} \equiv \sqrt{\Omega^2 + (\frac{\hbar\beta_{n,l}^2}{4mR^2})^2},$

$p_{n,l}^{(j)} = \int rdrd\varphi \xi_{n,l}^{*}(r,\varphi) r^{|l_j|} e^{il_j} e^{-r^2} / (2\omega^2)$ and $\chi, C_{n,m}^{(1,2)}$ are functions of system parameters $(\omega_j, \Omega, \delta_t, ...)$ and are approximately independent of time . we note on the origin of CR effect based one Eqs.(8), namely, the Rabi frequency is being shifted by $\beta_{n,l}$, and different roots of Bessel function induce oscillations of different frequencies, almost similar to CR effect of photon numbers in quantum optics.

B. Macroscopic self-trapping of OAM

Another interesting effects in our model systems the quantum self-trapping of orbital angular momentum. The same as previous section we limit our analysis to a stationary vortex core .with enough nonlinearity in our system we observe two induced effect is duo to interactions. The evolution of OAM in this case is shown in the first row of Fig. 3. The first effect is the non-sinusoidal a harmonic oscillations of OAM. The second effect is the self-trapping of OAM shown in panel (c), where the average of OAM on one component (here the exciton) is higher than the other.

C. OAM randomness of moving vortex

A very interesting regime of the vortex dynamics has been reported recently in exciton-polariton [18]. When a pulse carrying both orbital angular momentum and linear momentum (like Laugger-Guasian beam) is sent, the peculiarity of polariton dispersion induces splinting in exciton and photon wave packets, where the two packets move at different speeds. The dynamics results in some interesting features in output beams: vortex- anti vortex (pair) generation and annihilation plus OAM oscillations in time. Here we perform the similar analysis but in the presence of quantum confinement. We add an interesting feature to the moving vortex: the randomness of OAM. This implies the generation and annihilation of many pairs, excited due to orbital angular momentum discreteness. To explore such interesting dynamics we assume $(k_x = 1.2\mu m^{-1}, k_y = 0)$.in Fig. 3 and panels (d-f) the time variations of $\langle L_Z \rangle_{C,X}$ are shown. One can detect the CR effect, however it has a disorder trends compared to ordered patterns in stationary case (Fig. 1). More details of the dynamics is shown in Fig. 4. In panel (a), the OAM per particle $(\langle \tilde{L}_z \rangle)$ has time-varying behavior, however it has chaotic feature. In panel (b) the phase map of the

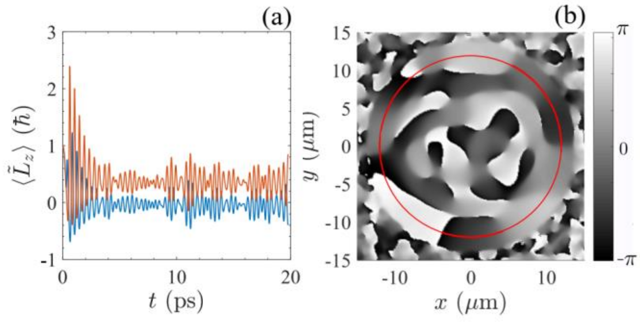

FIG. 4. (a) The regime of chaotic OAM in moving packets. The blue (red) lines are associated to photon (exciton) field. Panel (b) shows an example of phase map for photon field $(\arg[\psi_c])$.Due to motions of the packets and also discreteness of OAM, many pairs are being excited and annihilated during the dynamics. The red ring shows the border of the quantum well.

Photon filed is shown. In contrast to the stationary case, there are many pairs that are moving inside the quantum well, created and/or annihilated in time.

IV. CONCLUSIONS

In brief, we use some features of non-{classical states and integrate them with the orbital angular momentum (OAM). To this end, we utilize the physics of exciting polar ton in confined potential well in micro pillar. We show the regime of collapse and revival of OAM, rooted in quantum discreteness of OAM in circular quantum well. In nonlinear regime, the quantum self-trapping of OAM occurs: one component has more OAM compared to the other. We the packets carry linear momentum too, corresponding to regime of moving vortex; the OAM per particle shows a chaotic trend.

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