$F(x) = \lambda + x(1 - x)$ FUNKSIYA UCHUN MANDELBROT TO'PLAMINI TOPISH MASALASI

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ANNOTATSIYA

Ushbu maqolada matematikaning zamonaviy va qiziqarli yo'nalishlaridan biri bo'lgan dinamik sistema sohasiga oid bo'lgan $F(x) = \lambda + x(1 - x)$ funksiyaning dastlab qo'zg'almas nuqtalari, keyin Mandelbrot to'plamini, va nihoyat to'ldirilgan Julia, Julia va Fatu to'plamlarini topish masalasini ko'rib chiqdik.

Kalit so'zlar: Dinamik sistema, grafik, Mandelbrot to'plami, to'ldirilgan Julia, Julia to'plami, Fatu to'plami, qo'zg'almas nuqta, grafik analiz

ABSTRACT

In three articles, the properties of the fixed points of the function $F(x) = \lambda + x(1 - x)$ related to the dynamical system, which is one of the modern fields and interesting directions of mathematics, then Julia and Fatu sets, and finally we considered the problem of finding Mandelbrot sets.

Keywords: Dynamic system, graph, Julia set, Mandelbrot set, Fatu set, fixed point, graph analysis

KIRISH

Biz ushbu maqolada "Dinamik sistema" deb ataluvchi matematikaning zamonaviy va ajoyib bo'limiga doir misollar va ularning grafiglarini o'rganamiz. Dinamik sistema haqida dastlab qisqacha tushuncha keltirib o'tsak, koinotdagi yulduzlar va sayyoralar harakati dinamik harakatdir. Butun dunyoda ob-havo o'zgarishi, aholi sonining o'sishi va kamayishi dinamik harakatlarning yaqqol misollari hisoblanadi. Kimyo, biologiya va fizika, hattoki moliya sohasida dinamik harakatlarga ko'plagan misollar keltirish mumkin. Demak, dinamik sistemaga misollar ko'p.

ADABIYOTLAR TAHLILI VA METODOLOGIYASI

Dastlab misolimizni ishlash davomida foydalaniladigan to'plamlarga ta'riflarni qisqacha keltirib o'tmoqchimiz.

Dinamik sistemalarda turli xil trayektoriyalar mavjud bo'lib, shubhasiz, eng muhimlaridan biri qo'zg'almas nuqtadir.

Qo'zg'almas nuqta deganimiz $F(x_0) = x_0$ shartni qanoatlantiradigan x_0 nuqta bo'ladi va u simvolik ravishda quyidagicha belgilanadi:

$fix\{f\} = x_0$

1-ta'rif. Haqiqiy sonlar to'plamidan F(x) orqali trayektoriyalar cheksizlikka intiladigan nuqtalar to'plami Fatu to'plami deyiladi va F(F) = {x : F n(x) → ∞} kabi belgilanadi.

2-ta'rif. Haqiqiy sonlar to'plamidan F(x) orqali trayektoriyalar cheksizlikka intilmaydigan nuqtalar to'plami to'ldirilgan Julia to'plami deyiladi va K(F) = R\F(F) kabi belgilanadi.

3-ta'rif. To'ldirilgan Julia to'plamining chegarasi Julia to'plami deyiladi va J(F) = ∂K(F) kabi belgilanadi.

4-ta'rif. Agar to'plamning ixtiyoriy ikkita nuqtasini tutashtiruvchi kesma shu to'plamga tegishli bo'lsa, u holda to'plam chiziqli bog'lamli deyiladi.

5-ta'rif. Parametrga bog'liq funksiyalarda to'ldirilgan Julia to'plami bog'lamli bo'ladigan barcha parametrlar to'plami Mandelbrot to'plami M deyiladi.

Keling, $F(x) = \lambda + x(1 - x)$ funksiya dinamikasini o'rganamiz, buning uchun Julia, Fatu va Mandelbrot to'plamlarini topish masalasini ko'raylik. Dastlab Mandelbrot to'plamini topamiz:

$$F(x) = \lambda + x(1 - x) = \lambda + x - x^2$$

Funksiya grafikni tasavvur qilishimiz uchun dastlab parabola uchlarini aniqlab olamiz

$$x_0 = \frac{-1}{2 \cdot (-1)} = \frac{1}{2} \qquad y_0 = -\frac{1}{4} + \frac{1}{2} + \lambda = \frac{1}{4} + \lambda$$

Demak, bu parabolamiz shoxlari pasga qaragan va uchi $(\frac{1}{2}, \frac{1}{4} + \lambda)$ nuqtada joylashar ekan. Endi qo'zg'almas nuqtalarini aniqlab olishimiz kerak. Buning uchun quyidagi tenglamani yechamiz:

$$\lambda + x - x^2 = x$$
$$-x^2 + \lambda = 0$$
$$x^2 = \lambda$$
$$x = \pm\sqrt{\lambda}$$

Endi biz bu yechimlarning ichidan kattasini ya'ni $\sqrt{\lambda}$ ni tanlab olamiz. Va uni grafik analiz usulida ko'rib o'tganimizdek, shu nuqta y_0 nuqtadan oshib ketganda bu nuqtalar itaruvchi bo'la boshlaydi. Shuning uchun bu yechimni y_0 ga tenglaymiz:

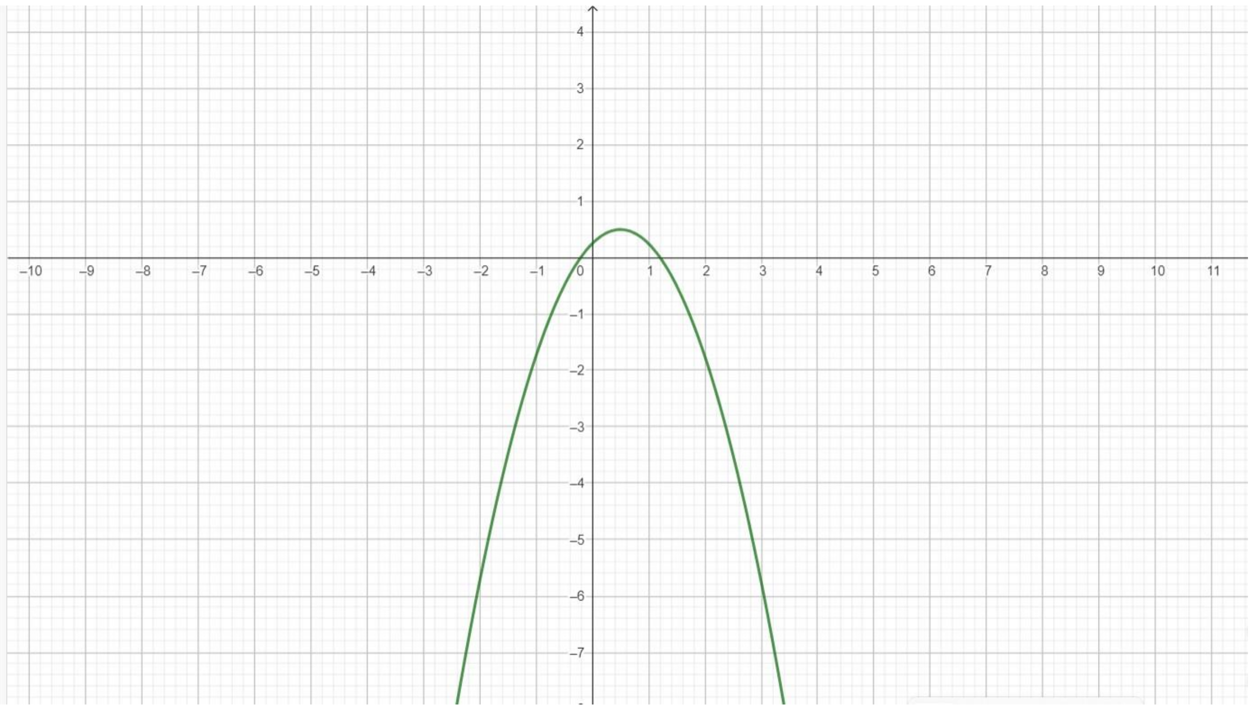
$$\sqrt{\lambda} = \frac{1}{4} + \lambda$$
$$\lambda = \frac{1}{16} + \frac{\lambda}{2} + \lambda^2$$

$$\left(\lambda - \frac{1}{4}\right)^2 = 0$$

$$\lambda = \frac{1}{4}$$

NATIJA

$\lambda \geq 0$ ekanligini hisobga olsak, Mandelbrot to'plami $\left[0, \frac{1}{4}\right]$ oraliq ekanligini topishimiz mumkin. Endi bu funksiya uchun Julia to'plami $\{-\sqrt{\lambda}, \sqrt{\lambda}\}$ va to'ldirilgan Julia to'plami esa $\left[-\sqrt{\lambda}, \sqrt{\lambda}\right]$ bo'ladi, chunki funksiya bu oraliqda grafik analiz bo'yicha cheksizlikka qarab ketmaydi. Endi Fatu to'plami esa $(-\infty, -\sqrt{\lambda}) \cup (\sqrt{\lambda}, \infty)$ bo'ladi ekan. Quyida ushbu funksiya grafigini $\left[0; \frac{1}{4}\right]$ oraliqdagi grafigini ko'ramiz:



XULOSA

Xulosa qilib aytadigan bo'lsak, $F(x) = \lambda + x(1 - x)$ funksiya uchun Mandelbrot to'plamini topish masalasi grafik analiz usulida yaqqol ko'rish mumkin. Agar $\lambda < 0$ bo'lganda, Mandelbrot to'plami mavjud emas. $\lambda > 0$ bo'lganda esa Mandelbrot to'plami $\left[0, \frac{1}{4}\right]$ oraliqda ekanligini ko'rishimiz mumkin. Xuddi shu kabi boshqa funksiyalarda ham umumiy holda Mandelbrot to'plamlarini topishimiz mumkin.

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